

HS215 COMPLEX VARIABLES AND SPECIAL FUNCTIONS

Course Description & Objectives:

The aim of this course is to introduce complex functions and their applications. Students learn about analytical functions, complex integration, classification of singularities etc. They would also learn conformal mappings. Some special functions and their applications will also be introduced.

Course Outcomes:

- I This course will help the student in analysis of real to complex numbers and apply them whenever the problem arises in real analysis and calculus.
- I The students will understand path and contour integrals
- I The student will be able to apply different theorems of integral formulae.
- I The student will be able to evaluate some standard integrals using contour integrals.
- I The student will be able evaluate the real integrals using special functions.

UNIT I - Analytic Functions:

Complex numbers, properties, (Brief discussion), Functions of complex variables, Limit and Continuity, Differentiability, Analytic functions, Cauchy – Riemann equations (without proof), Cauchy – Riemann equations in polar form (without proof), Orthogonal Curve, Harmonic functions, Conjugate harmonic functions, Constructions of conjugate harmonic functions, Milne Thomson method, Applications (Flow problems, Velocity potential etc.).

UNIT II - Complex Integration:

Line integral, properties of counter integrals, Cauchy's Integral theorem, Cauchy Integral formula and its generalization, Applications.

Convergence of series of complex terms, power series, region and radius of convergence, Taylor series, Maclaurin series and Laurent series.

UNIT III - Poles and Residues:

Singularity, Classification of Singularities, Pole at infinity. Zeros of analytic function, Residue of a pole, Residue at infinity, Residue theorem, Method of finding residues, Residue integrals, Evaluation of real definite integrals by

contour integration. Integration a round unit circle, of the

type $\int_q^{2p} f(\cos q, \sin q) dq$, of type $\int_q^{2p} \frac{p(x)}{q(x)} dx(p(x), q(x))$ polynomials,

integration on using rectangular contours, integration by indentation (functions having pole on-X axis).

UNIT IV - Conformal Functions:

Definition, conformal mapping by elementary functions, mapping $w=z^2$, transformations $w=e^z$, $w=\sin z$, $w=\cos z$, Joukvoski's transformation, Bilinear transformation.

UNIT V - Special Functions:

Gamma function, Beta function, Properties, Relation between Beta and Gamma functions, Application: Evaluation of integrals using Beta and Gamma functions.

Introduction series solutions of differential equations with variable coefficients, Bessel function and its properties.

TEXT BOOKS:

1. H.K.Das and Er.Rajnish Verma, Higher Engineering Mathematics, S.Chand & Co., New Delhi, 2011.
2. B.S.Grewal, "Higher Engineering Mathematics", 40th edition, Khanna Publishers, 2009.

REFERENCE BOOKS:

1. B.V.Ramana, "Higher Engineering Mathematics", 3rd Edition, Tata Mc Graw Hill Publishing Co., 2008.
2. R.K.Jain, SBK Iyengar, "Advanced Engineering Mathematics", 2nd Edition, Narosa Publishing House.