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Department of B.Sc(Mathematics,Statistics,Computer Science)

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A Field Project Report

On

Environmental studies - Air quality of area

Submitted by

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VADLAMUDI, GUNTUR-522213, AP, INDIA

February 2023

CERTIFICATE

This is to certify that the project report titled “Environmental Studies -Air quality of area submitted by CHINTHABATHINI VASANTHA BLESSY (221FM01002), DEVULAPALLI KOUNDINYA (221FM01003), KASU VEDANTH REDDY (221FM01005) is carried out as field project work under my supervision. I approve this field project work for submission towards partial fulfilment of the requirements and course work prescribed by 1st B.sc, VFSTR (Deemed to be University).

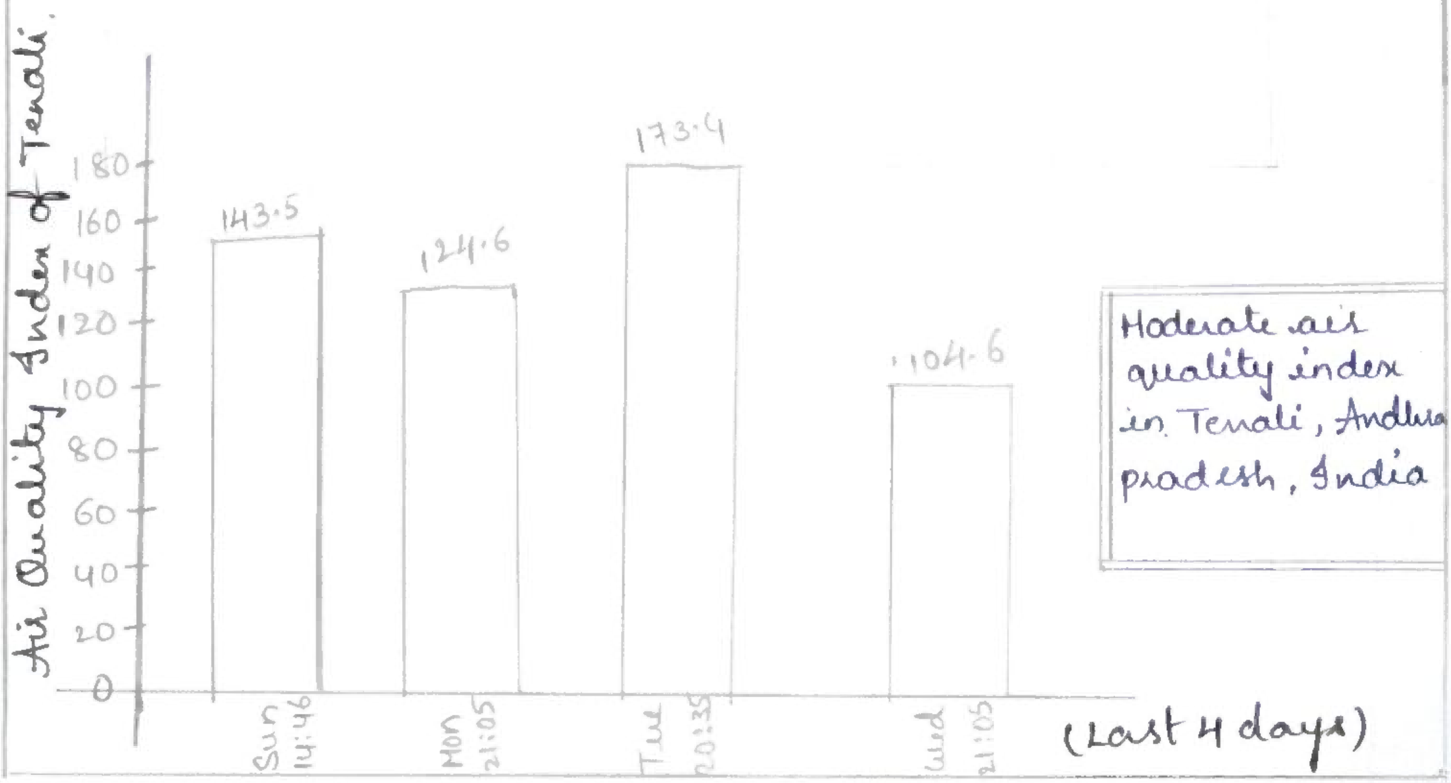
Project Guide

Head of the Department

1. what is the general air quality of the area?

A Our residential area is Tenali, Here the general air quality of the area is moderate. It consists of main pollutant : PM2.5 and others are.

O_3	AQI 28 Good.
PM2.5	AQI 104 poor
PM10	AQI 104 Moderate
Humidity	56.0%
Barometric pressure	1016.0 hpa.
Wind speed	7.34 m/s
Wind Direction	96.0 degrees



2 Is there any industry nearby? Is yes give the details.?

A No, there are no nearby industries in our area.

3 What are the noise levels of the area?

A In our are the noise levels are moderate, They ranges from 49.3 - 50.2. decibals in our area.

4 How is domestic solid wastes managed in the area? Give pictures.

A In our area domestic solid wastes are managed by the Tenali municipality corporation. They are managed in two types

ⓐ aerobic composting

ⓑ anerobic composting

Now, household kitchen organic wastes can be turned into manure through some simple methods.

Composting means turning of organic matter into nutrient - rich humus.

4a) Number of houses in the neighbourhood studied?

A) There are nearly 120 houses in the neighbourhood That I have studied

4b) What is the solid waste disposal method adopted by each family?

A) Here families are using methods like recycling, composting and energy from the waste. The least desirable waste management option is disposal.

They are using three strategies

* Reduction Strategies.
* Reuse strategies.
* Recovery strategies.

4c) Does Segregation of waste done prior to disposal by each family

A) Yes, the segregation of waste done prior to disposal by each family

4d) How much plastic is used by each of the family per day for various purposes.

A) Each family uses 7.6 gm of plastic per day for various purposes

4e): How does each family plan to reduce usage of plastics?

(A) Each family is planning to use ecofriendly bags instead of plastic bags.

5) Is there any pond | lake | water body | park or both present in the area?

A Yes, there is a pond in our area.

6 If a pond/ lake | water body | park is there describe it with a picture?

A) Pond consists of some plants in it. outersides of the pond there we have some place to play.
It is a aquatic habitate to fishes and ducks.
There we have different types of fishes. It totally covered with plants. In the middle of this, plant are there.

7 Describe any pollution problem present in the pond/ lake /water body | park?

A Some kind of water pollution is present in the lake of pond, such as plastic waste and the household items and many other wastes. Some of them threw pollutants in the pond.

8 Describe the biodiversity of the pond / lake / water body / park give pictures.

A Many ecosystems are linked by water and ponds have been found to hold a greater biodiversity

Generally, a pond is an area filled with water either natural (or) artificial, that is smaller than lake. ponds are small bodies of fresh water with shallow and still water, marsh and aquatic plants ponds can be created by wide variety of natural processes (Eg: on floodplains as cutoff river channels, by glacial processes, by peatland formation, in coastal dune systems, by beavers. They can be simply isolated depressions..

POND ECOSYSTEM

A Field Project Report

On

Rules of Inference and Logic Gates

(MATHEMATICS– FIELD PROJECT)

Report

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This is to certify that the project report titled "**Disjunctive normal form and Logic gate**" Submitted by **MAMIDI GEETHIKA (221FM01006), POTTURI SONU SAHANA (221FM01007), UMMANENI SIVA PRASAD (221FM01008)** is carried out as field project work under my supervision. I approve this field project work for submission towards partial fulfilment of the requirements and course work in prescribed for 1st B. Sc., VFSTR (Deemed to be University).

Project Guide

Head of Department

***Abstract*— A fundamental concept in propositional logic, it allows us to derive valid conclusions from given premises. Common rules include Modus Ponens, Modus Tollens, and Disjunctive Syllogism.**

A graphical method for simplifying Boolean expressions in digital logic design. It reduces terms and literals by grouping adjacent cells with ones in the truth table. K-maps aid in optimizing logic circuits for better performance and efficiency.

Keywords*— *Rule of Inference, Propositional Logic, Valid Conclusions, Modus Ponens, Modus Tollens, Disjunctive Syllogism, Karnaugh Map (K-map), Boolean Expressions, Truth Table, Digital Logic Design, Optimization.

I. INTRODUCTION

Rules of Inference**:** These guidelines help us draw valid conclusions from given premises in propositional logic. Examples include Modus Ponens and Modus Tollens.

Karnaugh Maps (K-maps)**:** These graphical tools simplify Boolean expressions by grouping adjacent cells in a grid-like format. They aid in minimizing logic functions and optimizing digital circuits.

RULE OF INFERENCE

The main function of logic key to provide rules of inference are principle of reasoning. The theory associated with such rules is known as inference theory. Because it is concerned with the inference.

Rule of Inference	Tautology	Name
P P →Q Q	[P ∧ (P →Q)] → Q	Modus Ponens
¬ Q P →Q ¬ Q	[¬ Q∧ (P →Q)] →¬P	Modus Tollens
P→Q Q→R ∴P→R	[(P→Q) ∧ (Q →R)] → [P→R]	Syllogism
P∨Q ¬P Q	[(P∨Q) ∧¬ P]→ Q	Disjunctive Syllogism
P ∴P∨Q	P → (P∨Q)	Addition
P∧Q ∴P	(P∧ Q) → P	Simplification
P Q ∴P∧Q	Q →(P ∧ Q)	Conjunction
¬ P ∨ R ∴Q ∨ R	[(P∨Q)∧(¬¬ P∨R)] → Q ∨ R	Resolution

Example:-

i. If it is snowing, then the roads are slippery.

ii. The roads are not slippery.

iii. Therefore, it is not snowing.

iv. If it is not snowing, then the roads are not slippery.

Ans:-

P – It is snowing.

Q – Roads are not slippery.

i. P → Q

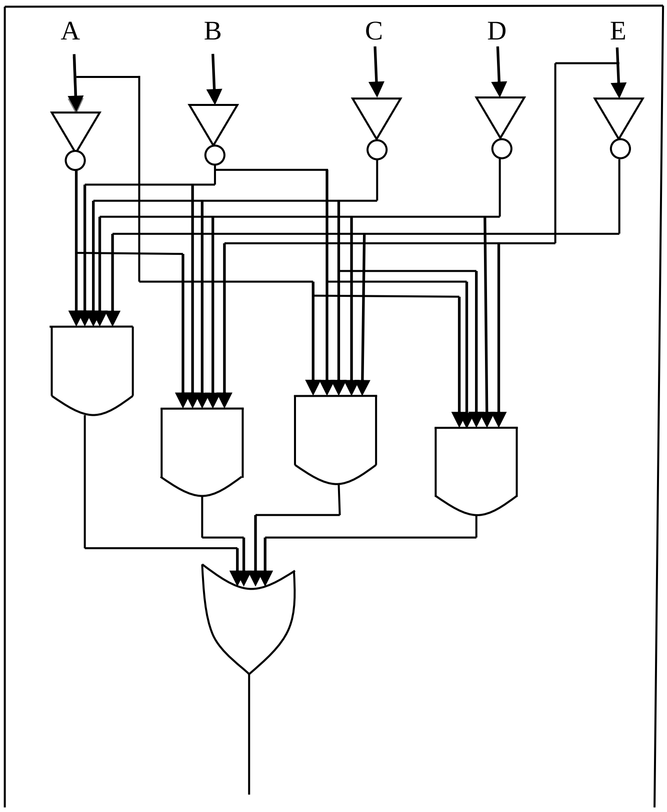
ii. ¬ Q

iii. ¬ P

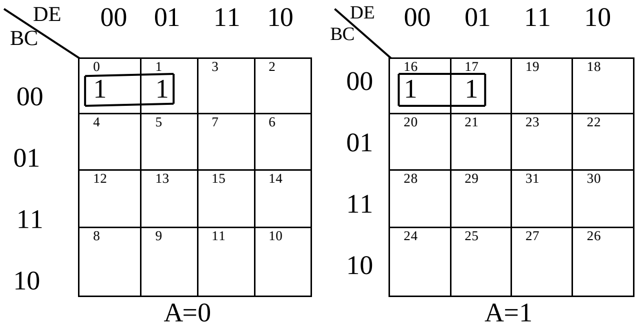
iv. ¬ P → ¬ Q

K-map

Q. f(A,B,C,D,E) = A'B'C'D'E' + A'B'C'D'E + AB'C'D'E' + AB'C'D'E



Construction of K-Map



From the K-map

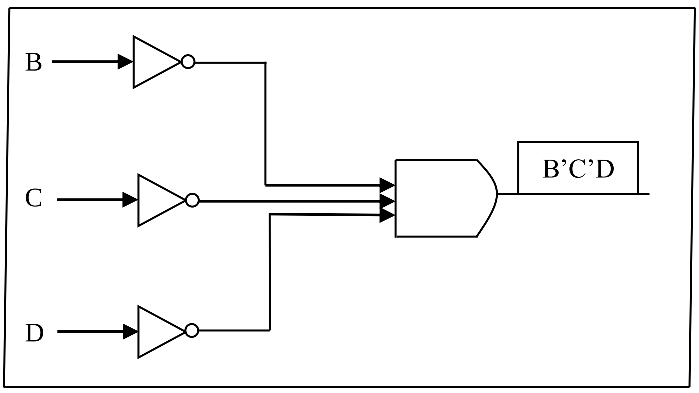
The prime applicant of G_1

B	C	D	E
0	0	0	0
0	0	0	1
B’	C’	D’	-

The simplified Boolean function

$f(A,B,C,D,E) = G_1$

$= B'C'D'$



CONCLUSION:

In our discussion, we explored two crucial concepts: Rules of Inference and Karnaugh Maps. Here’s a concise summary:

a. Rules of Inference:

These rules guide valid conclusions in propositional logic.

Examples: Modus Ponens (if A → B and A is true, then B is true) and Modus Tollens (if A → B and B is false, then A is false).

b. Karnaugh Maps (K-maps):

Graphical tools for Boolean expression simplification.

Visualize truth tables in a grid format.

Group adjacent cells (minterms or maxterms) to simplify expressions.

Crucial for digital circuit optimization.

II. REFERENCE

i. For Rules of Inference and Propositional Logic:

"Introduction to Logic" by Irving M. Copi and Carl Cohen.

"Symbolic Logic and Mechanical Theorem Proving" by Chin-Liang Chang and Richard Char-Tung Lee.

"A Concise Introduction to Logic" by Patrick J. Hurley.

ii. For Karnaugh Maps and Digital Logic Design:

"Digital Design: Principles and Practices" by John F. Wakerly.

"Fundamentals of Digital Logic with Verilog Design" by Stephen Brown and Zvonko Vranesic.

"Digital Logic Design: A Rigorous Approach" by Guy Even and Moti Medina.

Online Resources:

Stanford University's "Introduction to Logic" course available on platforms like Coursera or edX.

MIT OpenCourseWare offers various courses related to digital logic design and computer science.

The IEEE Xplore Digital Library contains numerous research papers and articles on topics related to logic and digital design.

Angle of intersection of two spheres

(MATHEMATICS– FIELD PROJECT)

Report

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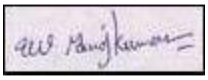
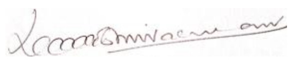




Project Guide **Head of the Department**

Abstract:

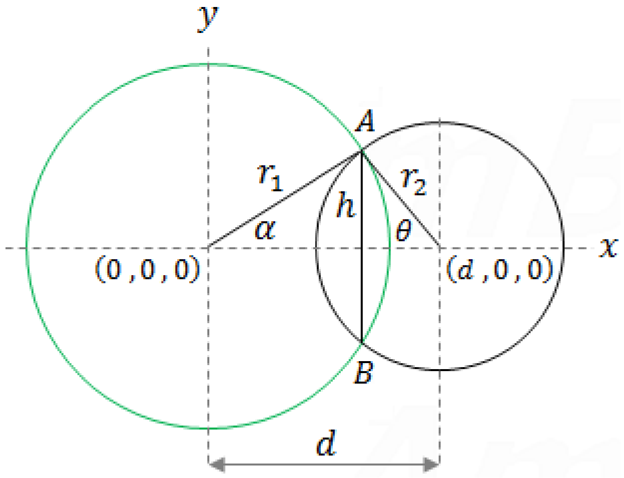
The **intersection** curve of **two spheres** always degenerates into the absolute conic and a circle. Therefore, the real **intersection of two spheres** is a circle. The plane determined by this circle is perpendicular to the line connecting the centers of the **spheres** and this line passes through the center of this circle.

INTRODUCTION:

A **sphere** is a geometrical object in three-dimensional space that is the surface of a ball (viz., analogous to the circular objects in two dimensions, where a "circle" circumscribes its "disk").

Like a circle in a two-dimensional space, a sphere is defined mathematically as the set of points that are all at the same distance *r* from a given point in a three-dimensional space.[2] This distance *r* is the radius of the ball, which is made up from all points with a distance less than (or, for a closed ball, less than *or equal to*) *r* from the given point, which is the center of the mathematical ball. These are also referred to as the radius and center of the sphere, respectively. The longest straight line segment through the ball, connecting two points of the sphere, passes through the center and its length is thus twice the radius; it is a diameter of both the sphere and its ball.

While outside mathematics the terms "sphere" and "ball" are sometimes used interchangeably, in mathematics the above distinction is made between a *sphere*, which is a two-dimensional closed surface embedded in a three-dimensional Euclidean space, and a *ball*, which is a three-dimensional shape that includes the sphere and everything *inside* the sphere (a *closed ball*), or, more often, just the points *inside*, but *not on* the sphere (an *open ball*). The distinction between *ball* and *sphere* has not always been maintained and especially older mathematical references talk about a sphere as a solid. This is analogous to the situation in the plane, where the terms "circle" and "disk" can also be confounded.



The equations of the spheres are given by:

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r_1^2 \qquad (1)$$

$$(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 = r_2^2 \qquad (2)$$

The outer intersection points of the two spheres forms a circle (AB) with radius h which is the base of two spherical caps.

To make calculations easier we choose the center of the first sphere at (0, 0, 0) and the second sphere at (d, 0, 0).

The distance d between the spheres centers is:

$$\boldsymbol{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Now we can find the angle α and θ by the cosine law:

$$\alpha = \cos^{-1}\frac{r_1{}^2 + d^2 - r_2{}^2}{2r_1 d} \qquad \theta = \cos^{-1}\frac{r_2{}^2 + d^2 - r_1{}^2}{2r_2 d}$$

Once we found the angle α we can find the intersection circle radius h.

$$h = r_1 \sin\alpha = \frac{\sqrt{4r_1{}^2 d^2 - (r_1{}^2 + d^2 - r_2{}^2)^2}}{2d}$$

The lapping volume between the two spheres contains two spherical caps the height of the spherical cap can be found by the same way as circular segment height.

The height of the spherical cap 1 is: $h_1 = r_1 * (1 - \cos\alpha)$

The height of the spherical cap 2 is: $h_2 = r_2 * (1 - \cos\theta)$

Remark: When α or θ is bigger than 90 degree then the spherical cap height is more than the radius and the volume of the cap is more than half sphere.

The volume of the lapping area which contains the two spherical caps is:

$$V = \frac{\pi h_1{}^2}{3}(3r_1 - h_1) + \frac{\pi h_2{}^2}{3}(3r_2 - h_2) = \frac{\pi}{3}\left(3r_1 h_1{}^2 - h_1{}^3 + 3r_2 h_2{}^2 - h_2{}^3\right)$$

The equation of a sphere can be described by the equation: $x^2+y^2+z^2 + Ax + By + Cz + D = 0$
The connections of the coefficients A, B, C and D to eq. (1) are:

$A_1 = -2x_1 \quad B_1 = -2y_1 \quad C_1 = -2y_1 \quad D_1 = x_1^2 + y_1^2 + z_1^2 - r_1^2$

If both spheres are given in this form the distance d between spheres centers is:

$$d = \frac{1}{2}\sqrt{(A_2 - A_1)^2 + (B_2 - B_1)^2 + (C_2 - C_1)^2}$$

Sphere 1 radius is: $$r_1 = \frac{1}{2}\sqrt{A_1{}^2 + B_1{}^2 + C_1{}^2 - 4D_1}$$

If we subtract the two spheres equations from each other we receive the equation of the plane that passes through the intersection points of the two spheres and contains the circle AB.
$\mathbf{X}^2(x_2 - x_1) + \mathbf{Y}^2(y_2 - y_1) + \mathbf{Z}^2(z_2 - z_1) + x_1^2 - x_2^2 + y_1^2 - y_2^2 + z_1^2 - z_2^2 - r_1^2 + r_2^2 = 0$
(3)

General plane equation is: $\mathbf{XA + YB + ZC + D = 0}$ where:

$\mathbf{A} = 2(x_2 - x_1) \quad \mathbf{B} = 2(y_2 - y_1) \quad \mathbf{C} = 2(z_2 - z_1) \quad \mathbf{D} = x_1^2 - x_2^2 + y_1^2 - y_2^2 + z_1^2 - z_2^2 - r_1^2 + r_2^2$

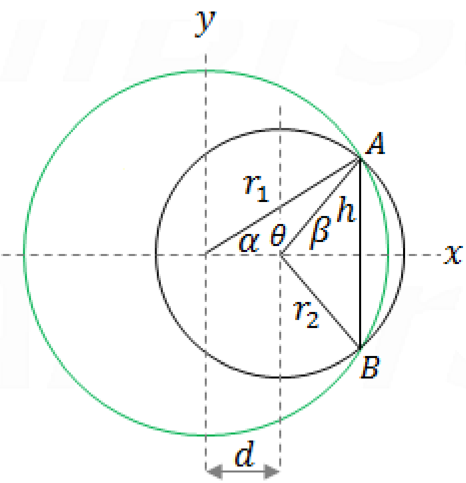
The equation of the line that connects the spheres centers is by parametric line equation

$x = x_1 + t(x_2 - x_1), \quad y = y_1 + t(y_2 - y_1), \quad z = z_1 + t(z_2 - z_1) \qquad (4)$

The center point of circle AB is located at the point of intersection of the parametric line connecting the spheres centers eq. (4) and the plane of the spheres intersection eq. (3) which also contains the circle AB.
By substituting eq. (4) into eq. (3) we can find the value of t which is:
$(x_1 + t\,x_2 - t\,x_1)A + (y_1 + t\,y_2 - t\,y_1)B + (z_1 + t\,z_2 - t\,z_1)C + D = 0$

$$t = \frac{x_1A + y_1B + z_1C + D}{A(x_1 - x_2) + B(y_1 - y_2) + C(z_1 - z_2)}$$

Substitute the value of t into eq. (4) to get the coordinate of the intersection circle (AB) center.



The type of intersection of two spheres depends on the size of the radii and the distance between the spheres centers.

Description	**Result**
$d < r_1 + r_2$ $d > \lvert r_1 - r_2 \rvert$	Conditions for intersection
$d > r_1 + r_2$	Two separate spheres
$d = r_1 + r_2$	Outer tangency
$d < \lvert r_1 - r_2 \rvert$	One sphere inside the other
$d = \lvert r_1 - r_2 \rvert$	inner tangency

A Field Project Report

on

RIGHT CIRCULAR CYLINDER WITH A GIVEN AXIS

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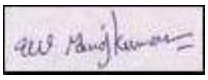
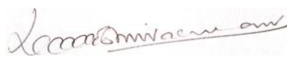
April 2023

<u>Certificate</u>

This is to certify the Project report titled **" RIGHT CIRCULAR CYLINDER WITH A GIVEN AXIS "** submitted by **AMARA VENKATA SESHA VEDASRITH SRIKAR(211FM01001),AMMISETTY PAVAN SAI KRISHNA (211FM01002),ARIKATLA YASWANTH DEVENDRA REDDY(211FM01004), BANDARU CHARAN SAI(211FM01005),BANDARU NANDINI(211FM01006)** is carried out as field project work under by supervisor.I approve this field project work for submission towards partial fulfillment of the requirements and course work in prescribed for B.Sc , VFSTR (Deemed to be University)





Project Guide **Head of the Department**

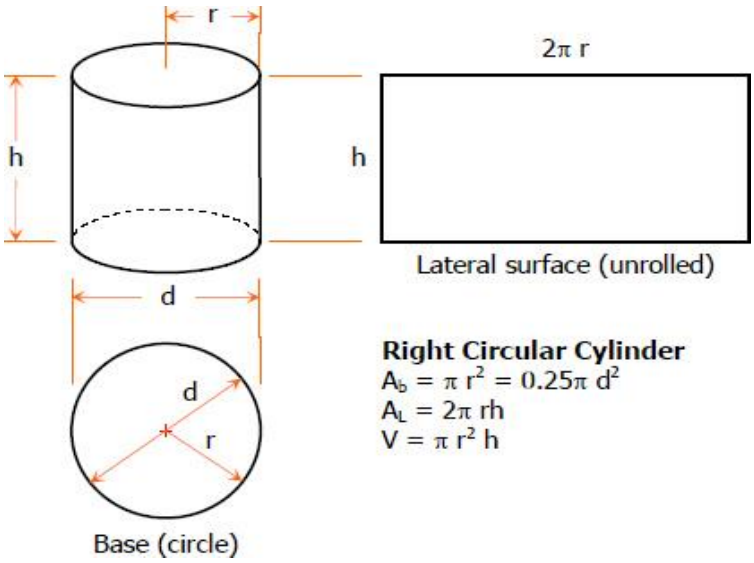
INTRODUCTION:A **right circular cylinder** is a cylinder that has a closed circular surface having two parallel bases on both the ends and whose elements are perpendicular to its base. It is also called **a right cylinder.** All the points lying on the closed circular surface is at a fixed distance from a straight line known as the axis of the cylinder. The two circular bases of the right cylinder have the same radius and are parallel to each other. It is one such geometric shape which is used frequently in real life. Basically, to derive the formulas for area and volume of the cylinder, the right cylinder is considered. There is also one more type of cylinder known as an oblique cylinder in 3D geometry, which is a special case.

Abstract: A cylinder whose bases are circular in shape and parallel to each other is called the right circular cylinder. It is a three-dimensional shape. The axis of the cylinder joins the centre of the two bases of the cylinder. This is the most common type of cylinder used in day to day life. Whereas the oblique cylinder is another type of cylinder, which does not have parallel bases and resembles a tilted structure.

Table of Content:

- Basics
- Properties
- Formula
- Solved Examples

Basics:A right circular cylinder is a cylinder whose base is a circle and whose elements are perpendicular to its base.



PROPERTIES OF RIGHT CIRCULAR CYLINDER

You must have learned about the properties of the cylinder before. Here, let us discuss the right circular cylinder properties.

- The line joining the centres of the circle is called axis.
- When we revolve a rectangle about one side as the axis of revolution, a right cylinder is formed.
- The section obtained on cutting a right circular cylinder by a plane, which contains two elements and parallels to the axis of the cylinder is the rectangle.
- If a plane cuts the right cylinder horizontally parallel to the bases, then its a circle.

RIGHT CIRCULAR CYLINDER FORMULA

A surface, which is generated by a line which intersects a fixed circle and is perpendicular to the plane of the circle is said to be a right circular cylinder. A right circular cylinder has two circular bases which are of the same radius and are parallel to each other. The formulas for surface area, curved or lateral surface area and volume of the right cylinder are discussed here.

Curved Surface Area

The surface area of a closed right circular cylinder is the sum of the area of the curved surface and the area of the two bases. The curved surface that joins the two circular bases is said to be the lateral surface of the right circular cylinder.

Lateral or Curved Area = $2 \pi r h$ square units

Total Surface Area

The sum of the lateral surface area and the base area of both the circles will give the total surface area of the right circular cylinder.

TSA = $2 \pi r (h + r)$ square units

Volume

The volume of a right cylinder is given by the product of the area of the top or bottom circle and the height of the cylinder. The volume of a right cylinder is measured in terms of cubic units.

Volume = Area of the circular bases x Height of the Right Cylinder

Volume = $\pi r^2 h$

Solved Examples

Example 1: Find the volume of a right cylinder, if the radius and height of the cylinder are 20 cm and 30 cm respectively.

Solution: We know,

Volume of a right cylinder = πr^2 h cubic units

Given, r = 20 cm h = 30 cm

Therefore, using the formula, we get;

Volume = $3.14 \times 20^2 \times 30$

= $3.14 \times 20 \times 20 \times 30$

= 37680

Hence, the volume of the given right cylinder is 37680 cm^3.

Example 2: The radius and height of a right cylinder are given as 5 m and 6.5 m respectively. Find the volume and total surface area of the right cylinder.

Solution: Given that, r = 5 m h = 6.5 m

We know, by the formula,

Volume of a right cylinder = πr^2 h cubic units

Therefore,

Volume = $3.14 \times 5^2 \times 6.5$

= $3.14 \times 25 \times 6.5$

= 510.25

Hence, the volume of the given right cylinder is 510.25 cubic m.

Now we know again, the total surface area of the right cylinder is given by;

TSA = Area of circular base + Curved Surface Area

TSA = $2 \pi r (h + r)$ square units

By putting the values of radius and height, we get;

TSA = 2 x π x 5 (6.5 + 5)

TSA = 2 x 3.14 x 5 x 11.5

TSA = 361.1 sq.m

Hence, the total surface area of the given right cylinder is 361.1 m^2.

Example 3:

Find the cost of digging a well **3m** in diameter and **24m** deep at the rate of **Rs.10** per cu.m.

Solution:

Given that

Diameter of the well =3m

∴ Radius of the well, r=1.5m

Depth of the well, h=24m

∴∴ Volume of the earth excavated =πr2h=227×(1.52)×24

=3.1416×2.25×24cu.m

=169.646cu.m

Now, cost per cu. m =Rs.10

∴ Total Cost=10×169.646=1696.46rupees

Example 4: The volume of a cylindrical ring is **800 cu.cm**. The radius of a cross section is **2cm**. Find the length of the ring.

Solution:

Given that

Radius of cross section, r=2cm

Volume, v=800cu.cm

Let the length of the ring be hcm

Now, Volume=area of cross – section × height

=πr2×h800=πr2×h

=227×4×h=227×4×h

∴ h=800×722×7=70011=63.6cm

Conclusion :

Given the axis of a right circular cylinder, you can determine its orientation, dimensions, and other properties. The axis defines the center line of the cylinder and its direction. With additional information such as the radius of the circular base or the distance between the bases, you can fully characterize the cylinder. If you need assistance with any specific aspect or calculation related to the cylinder

A Field Project Report

on

LENGTH OF THE PERPENDICULAR FROM THE GIVEN PLANE

Submitted by

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VADLAMUDI, GUNTUR – 52213, AP, INDIA

April 2023

Certificate

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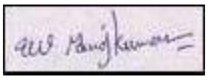
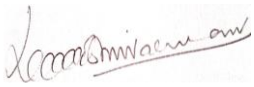




Project Guide **Head of the Department**

INTRODUCTION: The shortest distance of a point from a plane is said to be along the line perpendicular to the plane or in other words, is the perpendicular distance of the point from the plane. Thus, if we take the normal vector say ň to the given plane, a line parallel to this vector that meets the point P gives the shortest distance of that point from the plane. If we denote the point of intersection (say R) of the line touching P, and the plane upon which it falls normally, then the point R is the point on the plane that is the closest to the point P. Here, the distance between the point P and R gives the distance of the point P to the plane. In the upcoming discussion, we shall study about the calculation of the shortest distance of a point from a plane using the Vector method and the Cartesian Method..

ABSTRACT: In Euclidean geometry, the **distance from a point to a line** is the shortest distance from a given point to any point on an infinite straight line. It is the perpendicular distance of the point to the line, the length of the line segment which joins the point to nearest point on the line. The formula for calculating it can be derived and expressed in several ways.

Knowing the distance from a point to a line can be useful in various situations—for example, finding the shortest distance to reach a road, quantifying the scatter on a graph, etc. In Deming regression, a type of linear curve fitting, if the dependent and independent variables have equal variance this results in orthogonal regression in which the degree of imperfection of the fit is measured for each data point as the perpendicular distance of the point from the regression line.

Table of Content:

- Basic terms with formulae
- Solved Examples

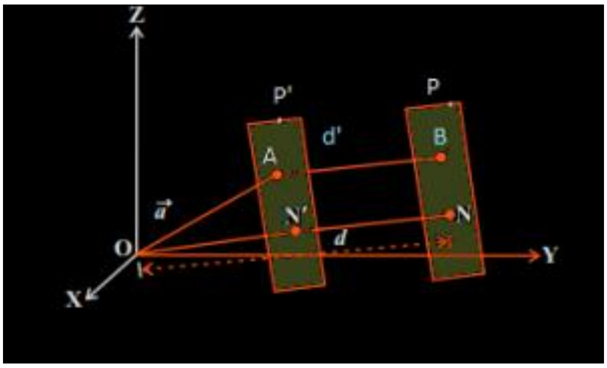
BASIC TERMS WITH FORMULAE: DISTANCE FROM POINT TO PLANE

Here's a quick sketch of how to calculate the distance from a point $P=(x1,y1,z1)$ to a plane determined by normal vector $N=(A,B,C)$ and point $Q=(x0,y0,z0)$. The equation for the plane determined by N and Q is $A(x-x0)+B(y-y0)+C(z-z0)=0$, which we could write as $Ax+By+Cz+D=0$, where $D=-Ax0-By0-Cz0$.

This applet demonstrates the setup of the problem and the method we will use to derive a formula for the distance from the plane to the point P.

Vector Form

Let us consider a point A whose position vector is given by â and a plane P, given by the equation, $\vec{r}.\vec{N} = d$



Here, N is normal to the plane P under consideration. Now, let O be the origin of the coordinate system being followed and P' another plane parallel to the first plane, which is taken such that it passes through the point A. Here, N' is normal to the second plane. The equation of the second plane P' is given by, $(\vec{r} - \vec{a}).\hat{N} = 0$

Or, $\vec{r}.\hat{N} = \vec{a}.\hat{N}$

We see that, the ON gives the distance of the plane P from the origin and ON' gives the distance of the plane P' from the origin. Thus, the distance between the two planes is given as, $ON - O\dot{N} = d' = \left|d - a.\hat{N}\right|$

This also given the perpendicular distance of the point A on plane P' from the plane P.

Thus we conclude that, for a plane given by the equation

$\vec{r}.\vec{N} = D$

, and a point A, with a position vector given by , the perpendicular distance of the point from the given plane is given by

$$d = \frac{|\vec{a}.\vec{N} - D|}{|\vec{N}|}$$

In order to calculate the length of the plane from the origin, we substitute the position vector by 0, and thus it comes out to be

$$d = \frac{|D|}{|\vec{N}|}$$

Cartesian Form

Let us consider a plane given by the Cartesian equation, Ax + By + Cz = D

And a point whose position vector is â and the Cartesian coordinate is,

$A(x_1, y_1, z_1)$

We can write the position vector as: $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

In order to find the distance of the point A from the plane using the formula given in the vector form, in the previous section, we find the normal vector to the plane, which is given as, $\vec{N} = A\hat{i} + B\hat{j} + C\hat{k}$

Using the formula, the perpendicular distance of the point A from the given plane is given as, $d = \frac{|\vec{a}.\vec{N} - d|}{|\vec{N}|}$

$$d = \left|\frac{(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}).(A\hat{i} + B\hat{j} + C\hat{k}) - D}{\sqrt{A^2 + B^2 + C^2}}\right|$$

$$d = \left|\frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}}\right|$$

This equation gives us the perpendicular distance of a point from a plane, using the Cartesian Method.

SOLVED EXAMPLES

Example 1: **Find the length and foot of the perpendicular from the point (7,14,5) to the plane 2*x*+4*y*−*z*=2.**

ANSWER

Let the foot of perpendicular from the point *P* to the given plane 2*x*+4*y*−*z*=2 be *Q*(*a*,*b*,*c*). So,

PQ=(*a*−7)*i*^+(*b*−14)*j*^+(*c*−5)*k*^

The normal vector of the given plane is,

N=2*i*^+4*j*^−*k*^

As *PQ*||*N*, so,

2*a*−7=4*b*−14=−1*c*−5=*λ*

a=2*λ*+7

$b=4\lambda+14$

$c=-\lambda+5$

Since $Q(a,b,c)$ lies on the given plane, so,

$2a+4b-c=2$

$2(2\lambda+7)+4(4\lambda+14)-(-\lambda+5)=2$

$\lambda=-3$

The coordinates of the foot of the perpendicular is $Q((2(-3)+7),(4(-3)+14),(-(-3)+5))=Q(1,2,8)$.

The length of the perpendicular is,

$PQ=(7-1)2+(14-2)2+(5-8)2$

$=189$

=321

<u>Example 2</u>**: The length of the perpendicular from the origin to the plane passing through the point *a* and containing the line *r*=*b*+*λc* is ?**

Solution: Given line is $r=b+\lambda c$

Let the plane equation be $(r-a)\cdot n\hat{}=0(1)$

as a passes through plane

$n\hat{}$ be normal vector to that plane

$n\hat{}\perp(a-b),n\hat{}\perp c$

We can write

$n=(b-a)\times c(2)$

Subtitute (2)*nd* equation in (1)*st* equation

$(r-a)\cdot((b-a)\times c)=0r\cdot((b-a)\times c)-a((b-a)\times c)=0r\cdot((b-a)\times c)=a(b\times c-a\times c)r\cdot((b-a)\times c)=a(b\times c)$

Above is the required plane equation

$r\cdot n=p$ is it is in the form then perpendicular distance from any vector (say $r1$) is

Formula: $|n||r1n-p|=L$

here $r1=0i\hat{}+0j\hat{}+0k\hat{}$

$p=[abc]n=[(b\times c)+(c\times a)]$

Substitute in the formula

$L=[(b\times c)+(c\times a)][abc]$

<u>Example 3</u>:

Find the length and the foot of perpendicular from the point (1,3/2,2)(1,3/2,2) **to the plane** 2x−2y+4z+5=0.

Solution:

Equation of the given plane is 2x−2y+4z+5=02x-2y+4z+5=0……….i

⇒n→=2iˆ−2jˆ+4kˆ⇒n→=2i^-2j^+4k^

So,the equation of line through (1,32,2)(1,32,2) and parallel to n→n→ is given by

x−12=y−3/2−2=z−24=λx-12=y-3/2-2=z-24=λ

⇒x=2λ+1,y=−2λ+32⇒x=2λ+1,y=-2λ+32 and z=4λ+2z=4λ+2

If this point lies on the given plane then

2(2λ+1)−2(−2λ+32)+4(4λ+2)5=02(2λ+1)-2(-2λ+32)+4(4λ+2)5=0

⇒4λ+2+4λ−3+16λ+8+5=0⇒4λ+2+4λ-3+16λ+8+5=0

⇒24λ=−12⇒λ=−12⇒24λ=-12⇒λ=-12

∴∴ Required foot of perpendicular

=[2×(−12)+1,2×(−12)+32,4×(−12)+2]i.e.(0,52,0)=[2×(-12)+1,2×(-12)+32,4×(-12)+2]i.e.(0,52,0)

∴∴ Required length of perpendicular=(1−0)2+(32−52)2+(2−0)2=SQRT[(1-0)2+(32-52)2+(2-0)2]

=6 units.

Conclusion :

The length of the perpendicular from a point to a plane is the shortest distance between that point and the plane. This distance can be calculated using the formula for the distance between a point and a plane:

Distance=$a2+b2+c2/ax0+by0+cz0+d|$

Where:

- (x_0, y_0, z_0) are the coordinates of the point,
- $ax+by+cz+d=0$ is the equation of the plane,
- (a, b, c) are the coefficients of the plane's normal vector..

By substituting the values of (x_0, y_0, z_0) into the equation of the plane and using the coefficients (a, b, c), you can calculate the distance.

.Remember to take the absolute value of the numerator to ensure the distance is positive.

A Field Project Report

on

Applications of PLANES-Shortest distance between planes

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<u>Certificate</u>

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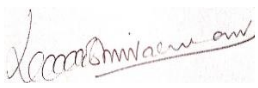


Project Guide   Head of Department

INTRODUCTION: A plane in R^3 is defined to be the locus of a point (x, y, z) satisfying a linear equation of the form ax + by+ cz = 0 where a, b, c are not all zero. It describes the equation of a plane, angle between the planes and perpendicular distance between a point and a plane and also the find the distance between two planes.

Abstract: According to the suitable formula, to Newton's law of cooling, the rate of loss of heat from a body is directly proportional to the difference in the temperature of the body and its surroundings.

Table of Content:

- Formula
- Derivation
- Solved Examples

Newton's law of cooling is given by, $dT/dt = k(T_t – T_s)$

Where,

- T_t = temperature at time t and
- T_s = temperature of the surrounding,
- k = Positive constant that depends on the area and nature of the surface of the body under consideration.

Newton's Law of Cooling Formula

Greater the difference in temperature between the system and surrounding, more rapidly the heat is transferred i.e. more rapidly the body temperature of body changes. Newton's law of cooling formula is expressed by,

$T(t) = T_s + (T_o – T_s)\ e^{-kt}$

Where,

- t = time,
- T(t) = temperature of the given body at time t,
- T_s = surrounding temperature,
- T_o = initial temperature of the body,
- k = constant.

Solved Examples

Example1: Find the distance between the parallel planes $2x - 2y - z + 3 = 0$ and $4x - 4y + 2z + 5 = 0$.

Find a point on the plane $2x - 2y - z + 3 = 0$ and the distance between the two parallel planes is the perpendicular distance from that point to the plane $4x - 4y + 2z + 5 = 0$.

The first plane meets the z-axis at the point $(0, 0, -3)$.

The length of the perpendicular from $(0, 0, -3)$ to the plane $4x - 4y + 2z + 5 = 0$ is $\pm$

$$\frac{4(0) - 4(0) + 2(-3) + 5}{\sqrt{4^2 + 4^2 + 2^2}} = \pm\frac{1}{6}$$

Hence the distance between the parallel planes is $\frac{1}{6}$

Example 2: Problem 1.7.4. *Find the equation of the plane passing through* (2, 2, 1) *and* (9, 3, 6)

and perpendicular to the plane $2x + 6y + 6z = 9$.

Solution. Equation of the plane passing through (2, 2, 1) is

$a(x - 2)b(y - 2) + c(z - 1) = 0$ · · · (1)

where a, b, c are the $d.r$ of the normal to the plane to determined.

Since (9, 3, 6) lies on this plane, we have $7a + b + 5c = 0$ · · · (2)

Since the plane (1) is perpendicular to $2x + 6y + 6z = 9$,

we have $2a + 6b + 6c = 0$ · · · (3)

Solving (2) and (3) we have $\frac{a}{-24} = \frac{b}{-32} = \frac{c}{40}$

Therefore $\frac{a}{3} = \frac{b}{4} = \frac{c}{-5} = k$ (say)

Therefore $a = 3k$, $b = 4k$, $c = -5k$.

Substituting in (1), we get $3(x - 2) + 4(y - 2) - 5(z - 1) = 0$ Therefore $3x + 4y - 5z - 9 = 0$.

Example 3: *Find the equation of the plane which passes through the point* (3, −2, 4) *and is perpendicular to the line joining the points* (2, 3, 5) *and* (1, −2, 3).

Solution. Since the plane is perpendicular to the line joining $A(2, 3, 5)$ and $B(1, -2, 3)$, the line AB is normal to the plane. The $d.r$ of the normal AB are 1,5,2.

Therefore the equation of the required plane is $1(x - 2) + 5(y + 2) + 2(z - 4) = 0$. That is, $x + 5y + 2z - 1 = 0$.

Example 4: *Find the equation of the plane through the intersection of the planes* $3x - y + 2z - 4 = 0$, $x + y + z - 2 = 0$ *and passing through the point* (2, 2, 1).

Solution. The required plane is $3x - y + 2z - 4 + a(x + y + z - 2) = 0$ where a is to be determined.

Since (2, 2, 1) lies on it, we have $6 - 2 + 2 - 4 + a(2 + 2 + 1 - 2) = 0$

$\therefore\ 2 + 3a = 0$. Hence $a = -\frac{2}{3}$.

Therefore the equation of the required plane is $3x - y + 2z - 4 - \frac{2}{3}(x + y + z - 2) = 0$

That is, $7x - 5y + 4z - 8 = 0$.

<u>Investigation</u>

For this exploration, Newton's Law of Cooling was tested experimentally by measuring the temperature in three beakers of water as they cooled from boiling. The purpose of this investigation was twofold. First I wanted to determine how well Newton's law of cooling fit real data. Second, I wanted to investigate the effect of changing the volume of water being cooled.

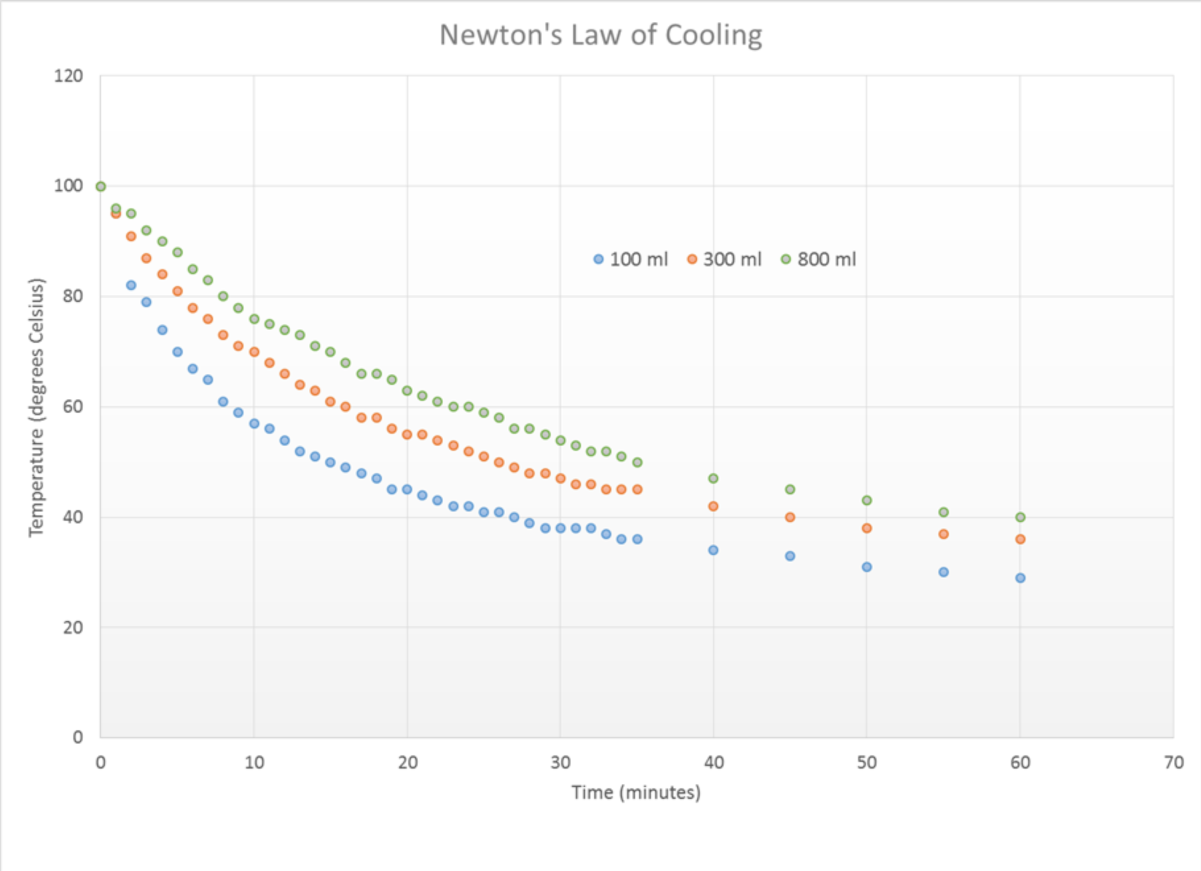
Three beakers of water were used for this experiment. The first held 100 ml of water, the second 300 ml, and the third 800 ml. All three beakers originally held water at 100°C. Each beaker had its own thermometer and the thermometers were kept in the beakers between measurements so there would be no temperature lag. The temperature of the water in each beaker was measured every minute, always in the same order. The ambient temperature for this investigation was 23°C. The experimental setup is shown below.

Conclusion: The temperature was measured every minute for 35 minutes and then every 5 minutes for the remainder of one hour. The following data was obtained.

Time (min)	100 ml Temperature °C	300 ml Temperature °C	800 ml Temperature °C
0	100	100	100
1	95	95	96
2	82	91	95
3	79	87	92
4	74	84	90
5	70	81	88
6	67	78	85
7	65	76	83
8	61	73	80
9	59	71	78
10	57	70	76
11	56	68	75
12	54	66	74
13	52	64	73
14	51	63	71
15	50	61	70
16	49	60	68
17	48	58	66

18	47	58	66
19	45	56	65
20	45	55	63
21	44	55	62
22	43	54	61
23	42	53	60
24	42	52	60
25	41	51	59
26	41	50	58
27	40	49	56
28	39	48	56
29	38	48	55
30	38	47	54
31	38	46	53
32	38	46	52
33	37	45	52
34	36	45	51
35	36	45	50
40	34	42	47
45	33	40	45
50	31	38	43
55	30	37	41
60	29	36	40

From this data, it can be observed that the water in the smaller beakers cooled more quickly than the water in the larger beakers. Below is a graph of the data.



In all of these cases, the experimental temperature fell more quickly at the beginning of the experiment than that predicted by the theoretical model and more slowly than predicted toward the end. The larger water sample followed the Newton's Law of Cooling model more closely than the smaller samples did. There are several explanations for this from a thermodynamics standpoint. Newton's Law of Cooling accounts primarily for conductive heat exchange and assumes that the only heat lost by the system to the surroundings is that due to the temperature difference. At temperatures near boiling, the rate of evaporation is high. The heat lost through the phase change is greater than the heat lost through convective heat exchange with the environment. Additionally, since the beakers were placed on a granite countertop, the heat lost through conduction with the countertop at the beginning of the experiment is significant and is higher than later on when the countertop has warmed up. If the countertop is now warmer than the surrounding air, the temperature gradient is not what it was assumed to be from the initial temperature measurement. Despite these complications, we conclude that Newton's Law of Cooling provides a reasonable approximation of the change in temperature for an object cooling in a constant ambient temperature.

Conclusion :

In conclusion, the concept of the shortest distance between planes has diverse applications across various disciplines, playing a crucial role in problem-solving, design, analysis, and optimization tasks. Its practical significance extends from engineering and physics to computer graphics and geographic information systems, contributing to advancements in technology, infrastructure, and scientific understanding.

A Field Project Report

on

Applications of Sphere-Equation of a Sphere through 4 points

Submitted by

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Project Guide

Head of the Department

INTRODUCTION: Newton's law of cooling describes the rate at which an exposed body changes temperature through radiation which is approximately proportional to the difference between the object's temperature and its surroundings, provided the difference is small.

Abstract: According to Newton's law of cooling, the rate of loss of heat from a body is directly proportional to the difference in the temperature of the body and its surroundings.

Table of Content:

- Formula
- Derivation
- Limitations
- Solved Examples

Newton's law of cooling is given by, $dT/dt = k(T_t - T_s)$

Where,

- T_t = temperature at time t and
- T_s = temperature of the surrounding,
- k = Positive constant that depends on the area and nature of the surface of the body under consideration.

Newton's Law of Cooling Formula

Greater the difference in temperature between the system and surrounding, more rapidly the heat is transferred i.e. more rapidly the body temperature of body changes. Newton's law of cooling formula is expressed by,

$T(t) = T_s + (T_o - T_s)\, e^{-kt}$

Where,

- t = time,
- T(t) = temperature of the given body at time t,
- T_s = surrounding temperature,

- T_o = initial temperature of the body,
- k = constant.

2.5 Sphere

Definition 2.5.1. *A **sphere** is the locus of a point in space which moves such that its distance from a fixed point is constant. The fixed point is called the centre of the sphere and the fixed distance is called the radius of the sphere.*

We now proceed to find several forms of the equation of a sphere.

1. Centre radius form

Theorem 2.5.2. *The equation of the sphere with centre $C(a, b, c)$ and radius r is given by $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$.*

Proof. Let $P(x_0, y_0, z_0)$ be any point on the sphere.
Hence $CP^2 = r^2$
Therefore $(x_0 - a)^2 + (y_0 - b)^2 + (z_0 - c)^2 = r^2$.
Therefore the locus of (x_0, y_0, z_0) is $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$. □

Corollary 2.5.3. *The equation of the sphere with centre origin and radius r is* $x^2 + y^2 + z^2 = r^2$.

2. General form of a sphere

Theorem 2.5.4. *The equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represents a sphere with centre $(-u, -v, -w)$ and radius $\sqrt{u^2 + v^2 + w^2 - d}$.*

Proof. The given equation can be written as
$(x + u)^2 + (y + v)^2 + (z + w)^2 = u^2 + v^2 + w^2 - d$.
This represents the locus of a point (x, y, z) which moves such that its distance from the point $C(-u, -v, -w)$ is equal to the constant $\sqrt{u^2 + v^2 + w^2 - d}$
Hence the given equation represents a sphere with centre $(-u, -v, -w)$ and radius $\sqrt{u^2 + v^2 + w^2 - d}$ □

2.6 Tangent Plane

Definition 2.6.1. *The straight line joining two points P and Q on a surface is called a chord of the surface. When Q moves along the surface and ultimately coincides with P the limiting position of PQ touches the surface at P and is called a tangent line of the surface.*
In the case of a sphere with centre C there are many tangent lines at a point P on it, all of them being perpendicular to the radius CP. All these tangents lie on the plane through P perpendicular to CP. This plane is called the **tangent plane** *of the sphere at P.*

Theorem 2.6.2. *The equation of the tangent plane to the sphere* $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ *at* $P(x_1, y_1, z_1)$ *is* $xx_1 + yy_1 + zz_1 + u(x + x_1) + v(y + y_1) + w(z + z_1) + d = 0.$

Note 2.5.6. The equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ can be denoted as $S = 0$ where $S \equiv x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d$.

3. Diameter form

Theorem 2.5.7. *The equation of the sphere described on the line joining the points* $A(x_1, y_1, z_1)$ *and* (x_2, y_2, z_2) *as diameter is given by* $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + (z - z_1)(z - z_2) = 0.$

Proof. Let $P(x, y)$ be any point on the sphere with AB as diameter.
Therefore the direction ratios of AP are $x - x_1, y - y_1, z - z_1$ and the direction ratios of BP are $x - x_2, y - y_2, z - z_2$.
Consider the circle passing through A, B and P. This circle also has AB as diameter and hence $\angle APB = 90^0$. [i.e] AP is perpendicular to BP.
Therefore $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + (z - z_1)(z - z_2)$=0
Since this is true for any point (x, y, z) on the sphere it represents the equation of the required sphere. □

Conclusion :

In conclusion, the applications of spheres and the equation of a sphere through four points are diverse and interdisciplinary, spanning mathematics, computer science, engineering, physics, and various applied fields. Their versatility makes them valuable tools for modeling, analysis, simulation, and problem-solving in a wide range of practical contexts.

A Field Project Report

on

APPLICATIONS OF CONES-EQUATION OF A CONE THOROUGH THE GIVEN VERTEX

Submitted by

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APRIL – 2023

CERTIFICATE

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Project Guide

Head of Department

INTRODUCTION:

In the previous unit we discussed a very commonly round three-dimensional object. In this unit we look at two more commonly found three-dimensional objects, namely, a cone and a cylinder. But, what you will see in this unit may surprise you—whal people usually call a cone or a cylinder are only portions of very particular cases of what mathematicians refer to as a cone or a cylinder.

We shall start our discussion on cones by defining them, and deriving their equations. Then wc shall concentrate on cones whose vertices are the origin. In particular, we will obtain the tangent planes of such cones.

The other surface that we will discuss in this unit is a cylindcr. We shall define a general cylindcr, and then focus on a right circular cylinder.

The contents in this unit are of mathematical interest, of course. But, they arc also of intercst to astronomers, physicists, engineers and architects, among others. This is because of the many applications that cones and cylinders have in various fields of science and engineering.

The surfaces that you will study in this unit are particular cases of conicoids, which you will study in the next block. So if you go through this unit carefully and ensure that you achieve the following objectives, you will find the next block easicr to understand.

In this short unit, our aim is to re-acquaint you with some two-dimensional geometry. We will briefly touch upon the distance formula and various ways of representing a line algebraically. Then we shall look at the polar representation of a point in the plane. Next, we will talk about what symmetry with respect to the origin or a coordinate axis is. Finally, we shall consider some ways in which a coordinate system can he transformed.

This collection of topics may seen random to you. But we have picked them according to our need. We will be using whatever we cover here, in the rest of the block. So, in later units we will often refer to a section, an equation or a formula from this unit.

You are probably familiar with the material covered in this unit. But please go through the following list of objectives and the exercises covered in the unit to make sure. Otherwise you may have some trouble in later units.

Abstract: According to Newton’s law of cooling, the rate of loss of heat from a body is directly proportional to the difference in the temperature of the body and its surroundings.

Table of Content:

- Formula
- Derivation

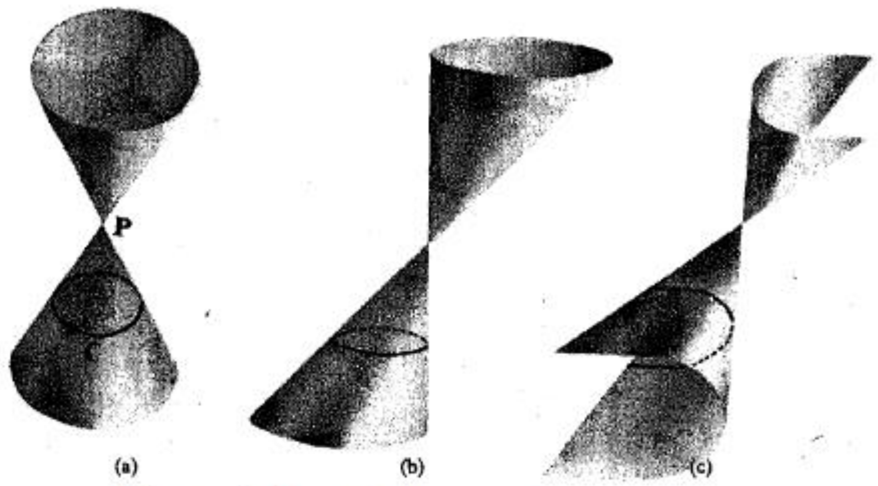

Fig.! : (a) A circular cone, (b) an elliptic cone, (c) a parabolic cone.

•

When you see an ice-cream cone, do you ever think that it is a set of lines? That is exactly what it is, as you will see in this section.

Definition: A cone is a set of lines that intersect a given curve and pass through a fixed point which is not in the plane of the curve. The fixed point is called the vertex of the cone, and the curve is called the base curve (or **directrix**) of the cone.

Each line that makes up a cone is called a generator of the cone.

Thus, we can also define a cone in the following way.

Definition: A cone is a surface generated by a line that intersects a given curve and passes through a fixed point which doesn't lie in the plane of the curve.

For example, in Fig. 1 (a), we give the cone generated by a line passing through the point P, and intersecting the circle C. The base curves of the concs in Fig. 1 (b) and Fig. 1 (c) are an ellipse and a parabola, respectively.

This theorem is the reason for an ellipse, parabola or hyperbola to be called a conic section (see Fig. 1 of Unit 3). It was proved by the Greek astronoiner Appolonius (approximately 200 B.C.). We will not give the proof here.

Now, according to Theorem 1 would you call a pair of intersecting lines a conic? If you cut a right circular cone by a plane that contains its axis, what will the resulting curve be? See Fig. 3.

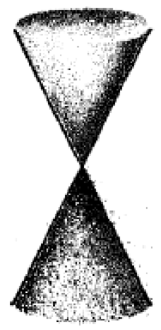

Fig. 3: A pair of intersecting lines is a conic section.

Let us now see how we can represent a cone algebraically. We shall first talk about a right circular cone, which we shall refer to as an **r.c.** cone.

So, let us take an r.c. cone. Let us assume that its vertex is at the origin, and its axis is the z-axis (see Fig. 4). Then the base curve, which is a circle of radius r (say), lies in a plane that is parallel to the XY-plane. Let this plane be z = k, where k is a constant. Then, any generator will intersect this curve in a point (a, b, k), for some a, b ∈ R. So the angle between the generator and the axis of the cone will be $\theta = \tan^{-1}\left(\frac{r}{k}\right)$, which is a constant.

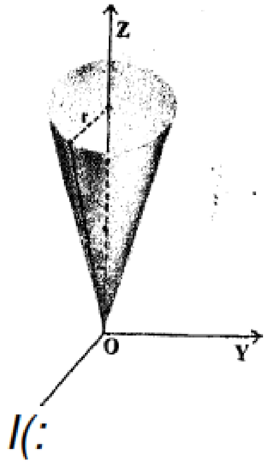

Fig. 4: A right circular cone with vertex at the origin and base curve $x^2 + y^2 = r^2$, $z = k$.

This is true for any generator of the cone.

Thus, every line that makes up the cone makes a fixed angle θ with the axis of the cone. This angle is called the **semi-vertical** angle (or generating angle) of the cone.

We can now define an r.c. cone in the following way.

Definition: A right circular **cone** is a **surface** generated by a line which passes through a fixed point (its **vertex**), and makes a constant angle with a fixed line through the fixed point.

Let us obtain the equation of an r.c. cone in terms of its semi-vertical angle. Let us assume that the vertex of the cone is O (0, 0, 0) and axis is the z-axis. (We can always choose our coordinate system in this manner.) Now take any point P (x, y, z) on the cone (see Fig. 5). Then, the direction ratios of OP are x, y, z, and of the cone's axis are 0, 0, 1. Thus, from Equation (9) of Unit 4, we get

$$\cos\theta = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

Thus, $x^2+y^2+z^2 = z^2 \sec^2\theta$

$\Rightarrow x^2+y^2 = z^2 \tan^2\theta$. ……(1)

(1) is called the standard form of the equation of a right **circular** cone.

Now, why don't you try the following exercises?

E1) Show that the equation of the r.c. cone with vertex at (a, b, c), axis $\frac{x-a}{\alpha} = \frac{y-b}{\beta} = \frac{z-c}{\gamma}$ and semi-vertical angle θ is

$[\alpha(x-a)+\beta(y-b)+\gamma(z-c)]^2 (\alpha^2+\beta^2+\gamma^2)\{(x-a)^2+(y-b)^2+(z-c)^2\}\cos^2\theta$ …… (2)

E2) Can you deduce (1) from (2) ?

E3) Find the equation of the r.c. cone whose axis is the x-axis, vertex is the origin and semi-vertical angle is $\frac{\pi}{3}$.

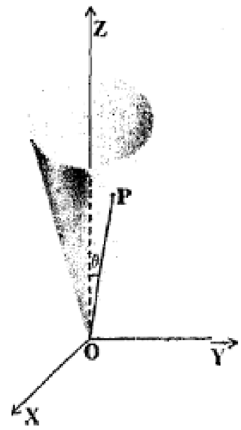

Fig. 5: $x^2 - y^2 = z^2$

Let us now look at a cone whose vertex is the origin. In this situation we have the following result.

Theorem 2: The equation of a cone whose base curve is a conic and whose vertex is (0, 0, 0) is a homogeneous equation of degree 2 in 3 variables.

Proof: Let us assume that the base curve is the conic

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, z = k.$

Any generator of the cone passes through (0, 0, 0). Thus, it is of the form

$$\frac{x}{\alpha} = \frac{y}{\beta} = \frac{z}{\gamma}. \qquad \ldots\ldots(3)$$

This line intersects the plane z = k at the point $\left(\frac{\alpha k}{\gamma}, \frac{\beta k}{\gamma}, k\right)$.

This point should lie on the conic. Thus,

$$\frac{k^2}{\gamma^2}(a\alpha^2 + 2h\alpha\beta + b\beta^2) + \frac{k}{\gamma}(2g\alpha + 2f\beta) + c = 0.$$

Eliminating α, β, γ from this equation and (3), we get

$$k^2\left(a\frac{x^2}{z^2} + 2h\frac{xy}{z^2} + b\frac{y^2}{z^2}\right) + k\left(2g\frac{x}{z} + 2f\frac{y}{z}\right) + c = 0.$$

$$\Rightarrow ax^2 + 2hxy + by^2 + 2gx\frac{z}{k} + 2fy\frac{z}{k} + \frac{cz^2}{k^2} = 0.$$

This is the equation of the cone. As you can see, it is homogeneous of degree 2 in the 3 variables x, y and z.

For example, the equation of the cone whose base curve is the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ in the plane z = 5, and whose vertex is the origin, is

$$\frac{x^2}{4} + \frac{y^2}{9} = \frac{z^2}{25}.$$

Do you see a pattern in the way we obtain the equation of the cone from the equation of the base curve? The following remark is about this.

Remark 2: To find the equation of the cone with vertex at (0, 0, 0) and base curve in the plane $Ax + By + Cz = D$, $D \neq 0$, we simply **homogenise** the equation of the curve, as follows. We multiply the linear terms by $\frac{Ax + By + Cz}{D}$, and the constant term by $\left(\frac{Ax + By + Cz}{D}\right)^2$; and we leave the quadratic terms as they are. The equation that we get by this process is a homogeneous equation of degree 2, and is the equation of the cone.

Let us look at a few examples of cones with their vertices at the origin.

Example 1: Show that the equation of the cone with the axes as generators is $fyz + gzx + hxy = 0$, where $f, g, h \in R$.

Solution: By Theorem 2, the equation of the cone is

$ax^2 + by^2 + cz^2 + 2fxy + 2gzx + 2hxy = 0$, for some $a, b, c, f, g, h \in R$.

Since the x-axis is a generator, (1, 0, 0) lies on it. Therefore, a = 0. Similarly, as it passes through (0, 1, 0) and (0, 0, 1), b = c = 0. So the equation becomes $fyz + gzx + hxy = 0$.

Example 2: Find the equation of the cone with vertex at the origin, and whose base curve is the circle $x^2 + y^2 + z^2 = 16$, $x + 2y + 2z = 9$.

l'heorem 3: A homogeneous equation of the second degree in 3 variables represents a cone whose vertex is at the origin.

Proof: Let the given equation be

$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0.$ ......(4)

Let P (a , β, γ) be a point on this surface and O the origin. Then OP is given by the equations

$\frac{x}{\alpha} = \frac{y}{\beta} = \frac{z}{\gamma} = r$ (say).

So any point on OP is (ra, rβ, ry). Since P lies on (4),

$a\alpha^2 + b\beta^2 + c\gamma^2 + 2f\beta\gamma + 2g\gamma a + 2h\alpha\beta = 0.$ ......(5)

Multiplying (5) throughout by r^2, we get

$a(r\alpha)^2 + b(r\beta)^2 + c(r\gamma)^2 + 2f(r\beta)(ry) + 2g(ry)(r\alpha) + 2h(m)(r\beta) = 0.$

Thus, (m, rβ, rγ) also lies on (4), for any r ∈ R. In particular, O lies on (4). So, the line OP lies on the surface given by (4). In other words, OP is a generator of (4). Thus, the surface (4) is generated by lines through the origin. Each of these lines will also pass through any curve obtained by intersecting (4) by a plane, and any'of these curves can be treated as a base curve. Thus, (4) represents a conc with the origin as vertex.

So, from what you have seen so far in this section; whenever you come across a homogeneous equation in 3 variables of degree 2, you know that it represents a cone.

Remark 3: If $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$, then (4) can be written as a product of two linear expressions. Thus, in this case (4) represents a pair of planes containing the origin. We shall consider this case as a degenerate cone, and any point on the line of intersection of the two planes can be considered as its vertex.

Using Theorem 3, we can show that if a, β, γ ∈ R, then a homogeneous equation in x − a, y − β, z − γ represents a cone with vertex at (a, β, y). (We shall discuss this kind of shifting in detail in Unit 7.)

Conclusion:

The equation of a cone through a given vertex serves as a versatile tool with applications across various disciplines, from mathematics and engineering to physics and beyond. Its significance lies in its ability to describe and analyze cone-shaped objects and phenomena in both theoretical and practical contexts.

A Field Project Report

on

Double Integration

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CERTIFICATE

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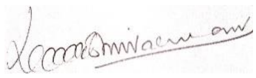

Project Guide | Head of Department

Abstract:

Double integration is a mathematical concept that extends the idea of single integration to two variables. It involves the process of finding the cumulative effect of changes in a function with respect to two independent variables over a region in a two-dimensional space. The concept is fundamental in calculus and has various applications in physics, engineering, economics, and other fields.

At its core, double integration is an abstraction that helps us understand and analyze quantities that depend on two variables simultaneously. It deals with functions of two variables and provides a way to calculate the total accumulated effect of changes in these variables over a specified region. The process involves iteratively integrating the function twice, first with respect to one variable and then with respect to the other.

Geometrically, double integration can be visualized as finding the volume under a surface in a three-dimensional space. The function being integrated represents the height of the surface at each point in the region. By integrating twice, we essentially find the total "volume" or accumulated effect of the function over the specified region.

INTRODUCTION:

DOUBLE INTEGRATION :

Double integration refers to the process of finding the integral of a function of two variables over a region in a two-dimensional plane. it involves integrating the function first with respect to one variable and then with respect to the other variable within a specified range or region. double integration is often used in calculus and applied mathematics to find areas, volumes, centroids, moments of inertia, and other properties of geometric shapes and regions

HOW TO FIND DOUBLE INTEGRATION :

To perform a double integration, follow these steps:

1) Identify the limits of integration for both variables (typically denoted as dx and dy).

2) Integrate the inner function with respect to one variable,treating the other variable as a constant.

3) Integrate the result obtained in step 2 with respect to the other variable, using the limits of integration determined in step 1

For example,if you have a function f(x,y) and you want to integrate it over a region R, the double integral is written as:

$$\iint f(x,y)dxdy$$

you would integrate f(x,y) first with respect to x, then integrate the result with respect to y, using the limits of integration for both variables.

- PROBLEM BASED ON AREA BY USING DOUBLE INTEGRATION :

1) ∬xydydx : R is the positive quadrant region bounded by x^2+y^2=a^2

Sol) Horizontal strip

x=0 to x=a to x^2+y^2=a^2

x^2=a^2 -y^2

x=√(a^2 -y^2)

Height

y=0 to y=a

= ∬xydydx

=∫ ∫ xydydx

=½∫ y[x^2/2] dy

=½∫ y[√(a^2 -y^2)]^2 dy

=½[a^2[y^2/2] -[y^4/4]]

=½[a^2[a^2/2] -[a^4/4]]

=½[a^2[2a^4-a^4/4]

=½[a^4/4]

=a^4/8

The area of the region Is a^4/8

2) ∫ ∫ $(x^2+y^2)dydx$

Sol) Limits vertical strip + length

$y=x$ to $y=a$

$x=0$ to $x=a$

Horizontal strip + length

$x=0$ to $x=y$

$y=0$ to $y=a$

∫ ∫ $(x^2+y^2)dydx$

∫ $(x^3/3+y^2x)$ dy

∫ $[y^3/3+y^3]$ dy

$=[y^4/12+y^4/4]$

$=A^4/12+a^4/4$

$=4a^4/12$

$=a^4/3$

The area enclosed is $a^4/3$

REAL LIFE APPLICATIONS OF DOUBLE INTEGRATION :

Double integration has numerous real-life applications across various fields. some examples are:

1. **PHYSICS:** Calculating the center of mass or centroid of a two-dimensional object with non-uniform density distribution. for example, finding the center of mass of a thin plate with varying density.
2. **ENGINEERING:** Determining the moment of inertia of irregularly shaped objects, which is crucial in structural engineering and mechanical design for analyzing the stability and strength of component

3. Geography and Cartography: Determining the area of irregularly shaped land masses or regions on maps, which is useful for land surveying, urban planning, and environmental studies.

4. Computer Graphics: Rendering three-dimensional objects on two-dimensional screen involves techniques like double integration to calculate shading, lighting, and other visual effects.

These are just a few examples, but double integration is a powerful tool with applications in a wide range of disciplines, including science, engineering, economics, and computer science.

CONCLUSION :

In conclusion, double integration, represented as $\iint f(x, y)\, dA$, is a fundamental technique in calculus for evaluating the integral of a function f(x, y) over a two-dimensional region D in the xy-plane. it can be expressed in either rectangular or polar coordinates:

1. Rectangular coordinates: $\iint f(x, y)\, da = \iint f(x, y)\, dxdy$

2. Polar coordinates: $\iint f(x, y)\, da = \iint f(r \cos\theta, r \sin\theta)\, r\, dr\, d\theta$

These formulas allow us to calculate various quantities such as area, volume, mass, and moments of inertia over the specified region D . Double integration is indispensable in physics, engineering, economics, and many other fields for solving problems involving two-dimensional quantities.

Conclusion:

The equation of a cone through a given vertex serves as a versatile tool with applications across various disciplines, from mathematics and engineering to physics and beyond. Its significance lies in its ability to describe and analyze cone-shaped objects and phenomena in both theoretical and practical contexts.

A Field Project Report

on

APPLICATIONS OF CONES-INTERSECTION OF A LINE AND QUADRATIC CONE

Submitted by

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Project Guide

Head of Department

INTRODUCTION: Newton's law of cooling describes the rate at which an exposed body changes temperature through radiation which is approximately proportional to the difference between the object's temperature and its surroundings, provided the difference is small.

Abstract: According to Newton's law of cooling, the rate of loss of heat from a body is directly proportional to the difference in the temperature of the body and its surroundings.

Table of Content:

- Formula
- Derivation
- Limitations
- Solved Examples

Newton's law of cooling is given by, $dT/dt = k(T_t - T_s)$

Where,

- T_t = temperature at time t and
- T_s = temperature of the surrounding,
- k = Positive constant that depends on the area and nature of the surface of the body under consideration.

Newton's Law of Cooling Formula

Greater the difference in temperature between the system and surrounding, more rapidly the heat is transferred i.e. more rapidly the body temperature of body changes. Newton's law of cooling formula is expressed by,

$T(t) = T_s + (T_o - T_s)\, e^{-kt}$

Where,

- t = time,
- T(t) = temperature of the given body at time t,
- T_s = surrounding temperature,
- T_o = initial temperature of the body,
- k = constant.

Theorem 4: If the cone $ax^2 + by^2 + cz^2 + 2fyx + 2gzx + 2hxy = 0$ has 3 mutually perpendicular generators, then $a + b + c = 0$.

Proof: Let the direction cosines of the three. mutually perpendicular generators be l_i, m_i, n_i, where $i = 1, 2, 3$. Since they are mutually perpendicular, we can rotate our coordinate system so that these lines become the new coordinate axes.

Then the direction cosines of the previous coordinate axes with respect to the new axes are $l_1, l_2, l_3; m_1, m_2, m_3$ and n_1, n_2, n_3, respectively.

So Unit 4 (Equations (3) and (10)) tell us that

$$\left.\begin{aligned} &l_1^2 + l_2^2 + l_3^2 = 1\\ &m_1^2 + m_2^2 + m_3^2 = 1\\ &n_1^2 + n_2^2 + n_3^2 = 1\\ &l_1m_1 + l_2m_2 + l_3m_3 = 0\\ &m_1n_1 + m_2n_2 + m_3n_3 = 0\\ &n_1l_1 + n_2l_2 + n_3l_3 = 0 \end{aligned}\right\} \quad \ldots\ldots(6)$$

Further, since the perpendicular lines arc generators of the cone, using E6 we get

$$al_1^2 + bm_1^2 + cn_1^2 + 2fm_1n_1 + 2gn_1l_1 + 2hl_1m_1 = 0$$
$$al_2^2 + bm_2^2 + cn_2^2 + 2fm_2n_2 + 2gn_2l_2 + 2hl_2m_2 = 0$$
$$al_3^2 + bm_3^2 + cn_3^2 + 2fm_3n_3 + 2gn_3l_3 + 2hl_3m_3 = 0$$

Adding these equations, and using (6), we gel $a + b + c = 0$.

Actually, the converse of this result is also true. The proof uses a fact that you have already seen in Fig. 3 in the case of an r.c. cone, namely,

x

e

any plane through the vertex of a cone intersects the cone in two lines, which may or may not be distinct.

The following result, which we shall not prove, tells us about the angle between the lines of intersection.

Example 3: Show chat if a + b + c = 0, then the conc

$C(x, y, z) \equiv ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$

has infinitely many sets of three mutually perpendicular generators.

Solution: Let $\frac{x}{\alpha} = \frac{y}{\beta} = \frac{z}{\gamma}$ be any generator of the conc. Then, by E6, we know that $C(\alpha, \beta, \gamma) = 0$. Therefore, using the fact that a+b+c = 0 and (8), we see that the plane $\alpha x + \beta y + \gamma z = 0$ intersects the cone in two mutually perpendicular generalors, say Land L'.

Now $\frac{x}{\alpha} = \frac{y}{\beta} = \frac{z}{\gamma}$ is normal to the plane $\alpha x + \beta y + \gamma z = 0$. Thus, it is perpendicular to both L and L'. Thus, these three lines form a set of three mutually perpendicular generators of the conc.

Note that we chose $\frac{x}{\alpha} = \frac{y}{\beta} = \frac{z}{\gamma}$ arbitrarily. Thus, for each generator chosen we get a set of three mutually perpendicular generators. Hence, the conc has infinitely many such sets of generators.

For convenience, we will write $C(x, y, z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy$.

Now consider the line $\frac{x - x_1}{\alpha} = \frac{y - y_1}{\beta} = \frac{z - z_1}{\gamma}$. Any point on this line is given by $(x_1 + r\alpha, y_1 + r\beta, z_1 + r\gamma)$, for some $r \in R$. Thus, the line will intersect the cone, if this point lies on the cone for some $r \in R$.

This happens if

$$a(x_1 + r\alpha)^2 + b(y_1 + r\beta)^2 + c(z_1 + r\gamma)^2 + 2f(y_1 + r\beta)(z_1 + r\gamma) + 2g(z_1 + r\gamma)(x_1 + ra) + 2h(x_1 + r\alpha)(y_1 + r\beta) = 0.$$

$$\Leftrightarrow r^2C(a, \beta, y) + 2r\{\alpha(ax_1 + hy_1 + gz_1) + \beta(hx_1 + by_1 + fz_1) + \gamma(gx_1 + fy_1 + cz_1)\} + C(x_1, y_1, z_1) = 0. \quad \ldots\ldots(9)$$

Now, if (x_1, y_1, z_1) doesn't lie on the cone, then (9) is a quadratic in r, and hence has two roots. Each root corresponds to a point of intersection of the line and the cone. Thus, we have just proved the following result.

Theorem 6: A straight line, passing through a point not on cone, meets the cone in at most two points.

Now suppose that the line $\frac{x - x_1}{\alpha} = \frac{y - y_1}{\beta} = \frac{z - z_1}{\gamma}$ is a tangent to the cone (4) at (x_1, y_1, z_1).

Then, since (x_1, y_1, z_1) lies on the cone, $C(x_1, y_1, z_1) = 0$. So (9) becomes

$$r^2C(\alpha, \beta, \gamma) + 2r\{\alpha(ax_1 + hy_1 + gz_1) + \beta(hx_1 + by_1 + fz_1) + \gamma(gx_1 + fy_1 + cz_1)\} = 0.$$

This equation must have coincident roots, since the line is a tangent to the cone at (x_1, y_1, z_1). The condition for this is

$$a(ax_1 + hy_1 + gz_1) + \beta(hx_1 + by_1 + fz_1) + y(gx_1 + fy_1 + cz_1) = 0. \quad \ldots\ldots(10)$$

So, (10) is the condition for $\frac{x - x_1}{a} = \frac{y - y_1}{\beta} = \frac{z - z_1}{\gamma}$ to be tangent to the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$.

Not that (10) is satisfied by infinitely many values of a, β, γ. Thus,

at each point of a cone we can **draw infinitely many** tangents to **the cone.**

Now, from Sec. 4.3.3, you know that (10) tells us that etch of these lines is perpendicular to the line with direction ratios

$ax_1 + hy_1 + gz_1, hx_1 + by_1 + fz_1, gx_1 + fy_1 + cz_1$.

Thus, the set of all the tangent lines at (x_1, y_1, z_1) is the plane

$$(x - x_1)(ax_1 + hy_1 + gz_1) + (y - y_1)(hx_1 + by_1 + fz_1) + (z - z_1)(gx_1 + fy_1 + cz_1) = 0$$

$$\Rightarrow x(ax_1 + hy_1 + gz_1) + y(hx_1 + by_1 + fz_1) + z(gx_1 + fy_1 + cz_1) = 0, \quad \ldots\ldots(11)$$

since $C(x_1, y_1, z_1) = 0$.

This plane is defined to be the **tangent** plane to the cone at (x_1, y_1, z_1).

Thus, (11) is the equation of the tangent plane at (x_1, y_1, z_1) to the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$.

There is a very **simple** working rule for writing (11).

Rule **of thumb**: To write the equation of the tangent plane at any point (a, β, γ) on the cone

Definition: A general second degree equation in three variables is an equation of the form

$$ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d = 0, \quad \text{.... (1)}$$

where a, b, c, d, f, g, h, u, v, w are real numbers and at least one of a, b, c, f, g, h is non-zero.

Note that if we put either z = k, a constant, x = k or y = k, in (1), then the equation reduces to a general second degree equation in two variables, and therefore, represents a conic.

Now we shall see what a general second degree equation in three variables represents. Let us first consider some particular cases of (1).

Case 1 : Suppose we put a = b = c = 1 and g = h = f = 0 in (1). **Then we get** the equation

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \quad \text{.... (2)}$$

Does this equation seem familiar to you? In Unit 6 you saw that if $u^2 + v^2 + w^2 - d > 0$, then (2) represents a sphere with centre (– f, –g, –h) and radius $\sqrt{u^2+v^2+w^2-d}$.

Case 2 : Suppose we put u = v = w = d = 0 in (1), then we get

$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0.$

What does this **equation** represent? You know from Unit 6 that this equation represents a cone.

Case 3 : If we put a = b = 1, h = 0 and z = k in (1), then it reduces to

$$x^2 + y^2 + 2ux + 2vy + d = 0, z=k \quad \text{.... (3)}$$

This represents a right circular cylinder (see Unit 6, Sec.6.4).

Similarly you can see that if we put x = k or y = k and a = b = 1, h = 0, then again (3) represents a cylinder.

We will discuss the surfaces represented by (1) in detail in the next unit.

The particular cases 1, 2 and 3 suggest that the points whose coordinates satisfy (1) lie on a surface in the three-dimensional system. Such a surface is called a conicoid or a quadric. Algebraically, we define a conicoid as follows:

Definition : A conicoid (or **quadric surface**) in the XYZ-coordinate system is the set S of points (x, y, z) $\in R^3$ that satisfy a general second degree equation in three variables.

So, for example, if

$F(x,y,z) \equiv ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$

is the second degree equation, then

$S = \{(x,y,z) \in R^3 \mid F(x,y,z)=0\}$

Note that S can be empty. For example, if $F(x,y,z) \equiv x^2+y^2+z^2+1=0$, then

$S = \{(x,y,z) \mid F(x,y,z) = 0\} = \phi$, the empty set.

In such cases we call the conicoid an imaginary conicoid.

Since the above expression is very lengthy, for convenience we often denote this conicoid by F (x,y,z,) = 0.

Note : In future, whenever we use the expression F(x, y, z) = 0, we will mean the equation (1).

In the last unit you saw that a point P is called a centre of a conicoid F (x,y,z)=0 if its coordinates satisfy a ssystem of linear equations (see Equations (18) of Unit 7). In this unit we define a centre geometrically and then see the relationship between the geometrically and analytical defections. Let us consider the conicoid S given by
$ax^2 by^2 cz^2 + d=0, abc + 0$

Let $P(x_1,y_1,z_1)$ be a point on the conicoid. Then you can see that $P' (-x_1,-y_1,-z_1)$ also lies on the conicoid. This means that S is symmetric about the origin O. Because of this property O is called tlie centre of the conicoid (see Fig.1)

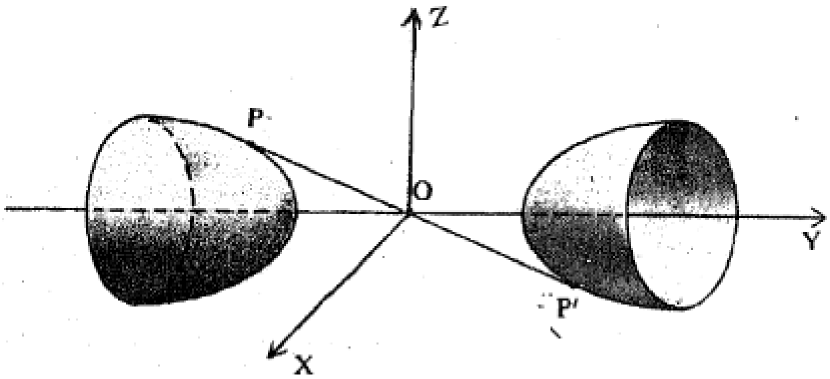

Fig. 1 : The pair of points P and P' are symmetric about the centre O of the conicold,

Definition : A conicoid S is called **symmetric with respect to a point** P if, when tlie origin is shifted to P, the transformed conicoid S is syininetric with respect to tlie origin.
Let us formally define a centre. To do so let us first see what we mean by symmetry
Definition : A point P is called a **centre of a conicoid S** if S is symmetric with respect to P.

Using the definition above we can easily see that the origin O(0,0,0) is a centre of the sphere.

Let us consider an example now.

Example 1: Show that the origin is the only centre of the cone
$ax^2 + by^2 + cz^2 = 0, abc \neq 0$

Solution : From the definition you can see that the origin is a centre of the sphere. Now let us take another point (x_0,y_0,z_0) which is 11011-zero. Shifting the origin to (x_0,y_0,z_0), we get the relationship.
$x = x' + x_0, y = y_0, z = z' z_0,$

wliere (x', y', z') denote the coordinates in the new system. Substituting tlie equations above in the equation of the cone, we get

$a(x' + x_0)^2 + b(y' + y_0)^2 + c(z' + z_0)^2 = 0$
$\Rightarrow ax'^2 + by'^2 + cz'^2 + 2[ax' x_0 + by'y_0 + cz'z_0] + ax_0^2 + by_0^2 + cz_0^2 = 0$
$\Rightarrow ax'^2 + by'^2 + cz'^2 + 2[ax' x_0 + by'y_0 + cz'z_0] = 0$

This is the transformed equation of the cone. Because of the non-zero linear summand (inside brackets) of this equation, we see that (0, 0, 0) is the only point about which the transformed cone is symmetric. Hence the origin (0, 0, 0) is the only centre of the cone.
$ax^2 + by^2 + cz^2 = 0, abc \neq 0.$

Conclusion:

The intersection of a line and a quadratic cone is a fundamental concept with diverse applications across various disciplines. Its study not only deepens our understanding of geometry and algebra but also enables us to tackle real-world problems with mathematical precision and insight.

A Field Project Report

on

RIGHT CIRCULAR CYLINDER

Submitted by

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NAAC A+ Accredited

(ACCREDITED BY "NAAC A+" GRADE)

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APRIL – 2023

CERTIFICATE

This is to certify that the project report titled "**RIGHT CIRCULAR CYLINDER"** Submitted by **DARAPUNENI SAI CHAITANYA (211FM01044), KONAKALA BHARGAV (211FM01046), KOTA UDAY AKHIL (211FM01047), PERUMALLA GEETHA KRISHNA (211FM01048), SHAIK AFRIN (211FM01049)** is carried out as field project work under my supervision. I approve this field project work for submission towards partial fulfilment of the requirements and course work in prescribed for B.Sc., VFSTR (Deemed to be University).

Project Guide   Head of Department

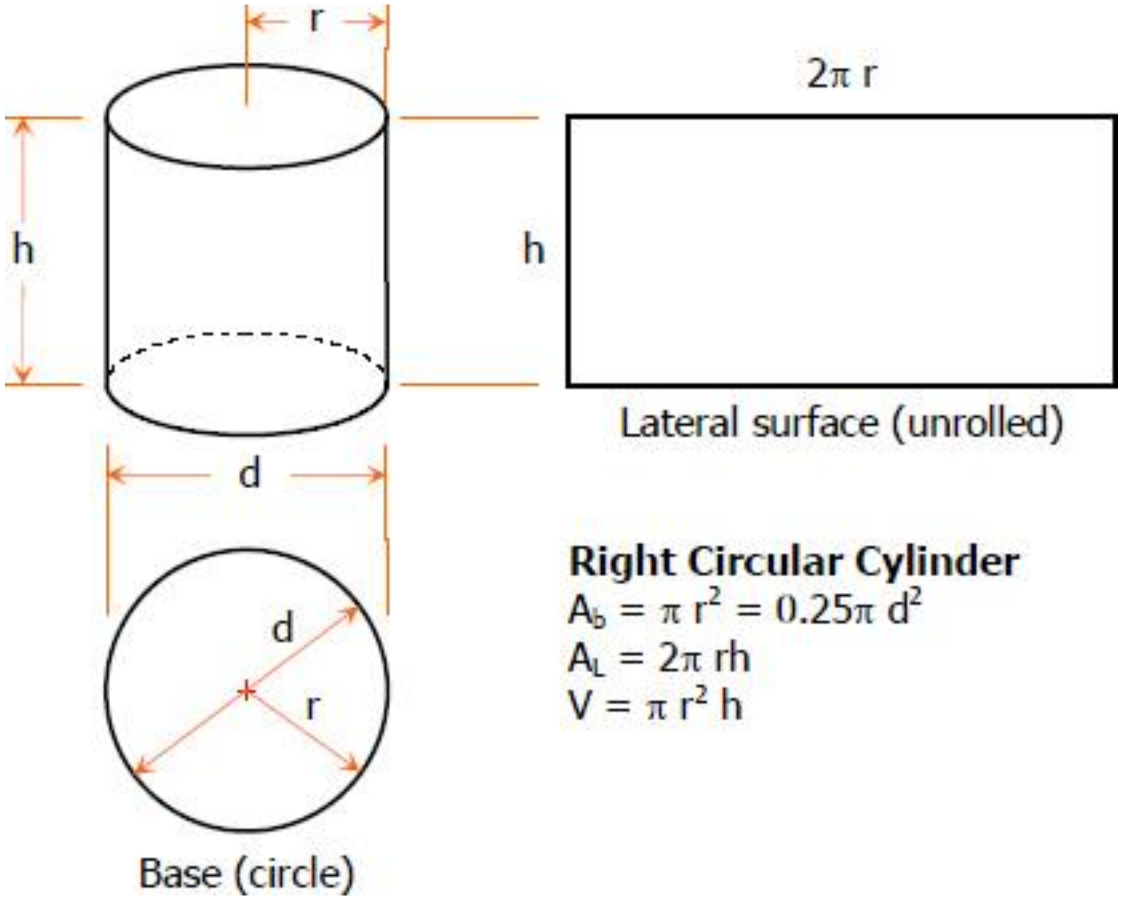
INTRODUCTION:

An attempt is made to study the equations of different geometrical structures and how the shapes will be visible for different values of the parameters of the shapes of these structures.

Abstract:

The Right Circular Cylinder

A right circular cylinder is a cylinder whose base is a circle and whose elements are perpendicular to its base.



Properties of a Right Circular Cylinder

1. The axis of a right circular cylinder is the line joining the centers of the bases.
2. For any oblique or non-oblique sections which do not pass any one base, the center of which is at the axis.
3. A right circular cylinder can be formed by revolving a rectangle about one side as axis of revolution.

4. Every section of a right circular cylinder made by a cutting plane containing two elements and parallel to the axis is a rectangle.

Formulas for Right Circular Cylinder

Area of the base, A_b

$$A_b=\pi r^2 \quad Ab=\pi r2$$
$$A_b=\pi 4 d^2 \quad Ab=\pi 4 d2$$

Lateral Area, A_L

$$A_L=2\pi rh \quad AL=2\pi rh$$
$$A_L=\pi dh \quad AL=\pi dh$$

Volume, V

$$V=A_b h \quad V=Abh$$
$$V=\pi r^2 h \quad V=\pi r2h$$
$$V=\pi 4 d^2 h \quad V=\pi 4d2h$$

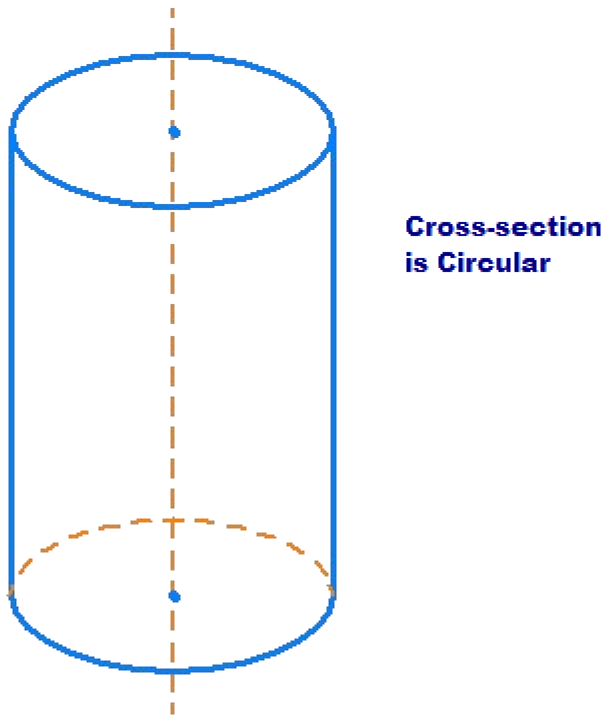
Total Area, A_T

Total area (open both ends), $A_T=A_L \quad AT=AL$

Total Area (open one end), $A_T=A_b+A_L \quad AT=Ab+AL$

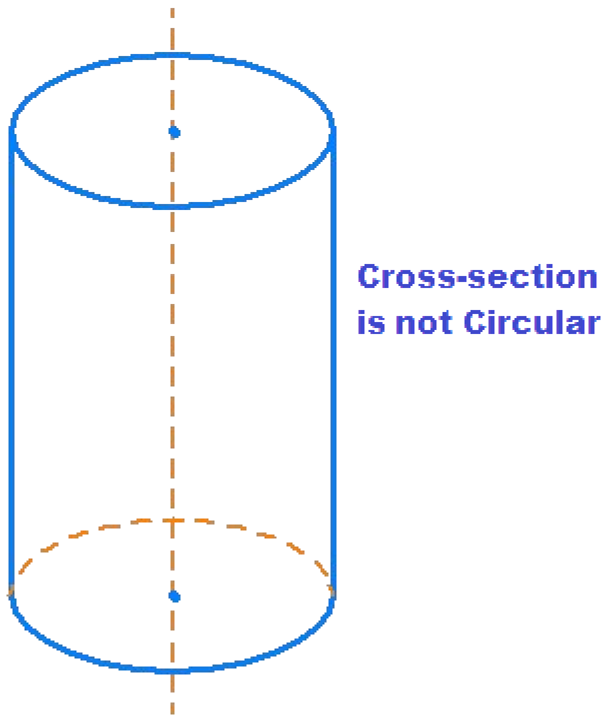
Total Area (closed both ends), $A_T=2A_b+A_L$

he following figure shows a right circular cylinder:

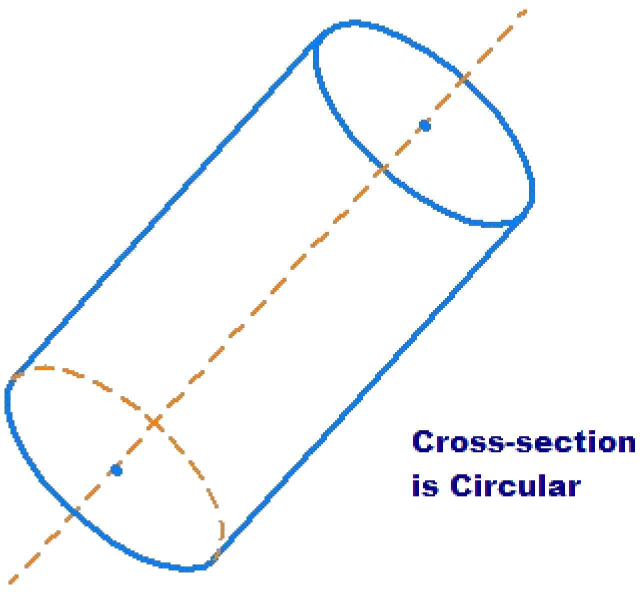


Note that the *cross-section* of a right circular cylinder is a circle, and the *axis* of the cylinder is perpendicular to the base.

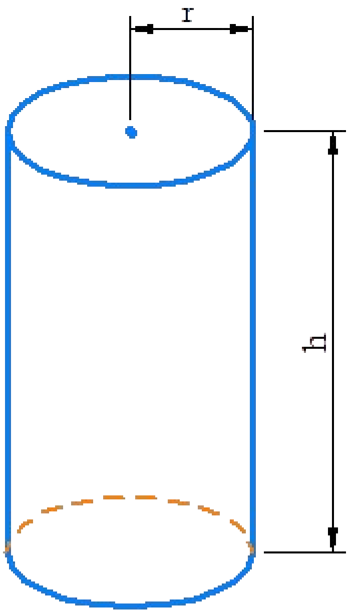
Below is an example of a right cylinder which is not circular (the cross-section is not a circle):



Below is an example of a circular cylinder which is not right (the cross-section is circular, but the axis is not perpendicular to the base):



From now on, we will only discuss right circular cylinders, and refer to them as simply cylinders. Observe that a cylinder's dimensions can be specified by two parameters: the radius r of the flat face (or the base), and its height h, as shown below.



Cylinder

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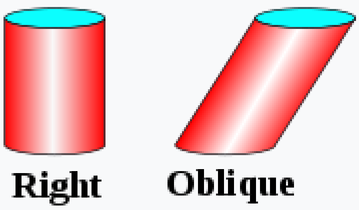
For other uses, see Cylinder (disambiguation).

An empty tin can

A **cylinder** (from Greek κύλινδρος – *kulindros*, "roller", "tumbler"[1]) has traditionally been a three-dimensional solid, one of the most basic of curvilinear geometric shapes. It is the idealized version of a solid physical tin can having lids on top and bottom.

This traditional view is still used in elementary treatments of geometry, but the advanced mathematical viewpoint has shifted to the infinite curvilinear surface and this is how a cylinder is now defined in various modern branches of geometry and topology.

The shift in the basic meaning (solid versus surface) has created some ambiguity with terminology. It is generally hoped that context makes the meaning clear. Both points of view are typically presented and distinguished by referring to *solid cylinders* and *cylindrical surfaces*, but in the literature the unadorned term cylinder could refer to either of these or to an even more specialized object, the *right circular cylinder*.

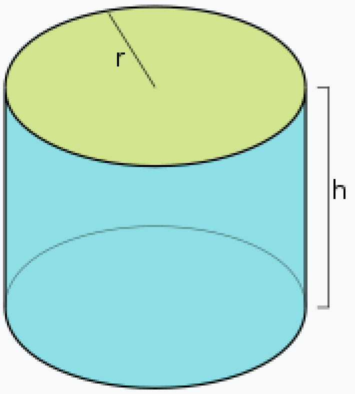


A right and an oblique circular cylinder

A solid bounded by a cylindrical surface and two parallel planes is called a (solid) *cylinder*. The line segments determined by an element of the cylindrical surface between the two parallel planes is called an *element of the cylinder*. All the elements of a cylinder have equal lengths. The region bounded by the cylindrical surface in either of the parallel planes is called a *base* of the cylinder. The two bases of a cylinder are congruent figures. If the elements of the cylinder are perpendicular to the planes containing the bases, the cylinder is a *right cylinder*, otherwise it is called an *oblique cylinder*. If the bases are disks (regions whose boundary is a circle) the cylinder is called a *circular cylinder*. In some elementary treatments, a cylinder always means a circular cylinder.[2]

The *height* (or altitude) of a cylinder is the perpendicular distance between its bases.

The cylinder obtained by rotating a line segment about a fixed line that it is parallel to is a *cylinder of revolution*. A cylinder of revolution is a right circular cylinder. The height of a cylinder of revolution is the length of the generating line segment. The line that the segment is revolved about is called the *axis* of the cylinder and it passes through the centers of the two bases.



A right circular cylinder with radius *r* and height *h*

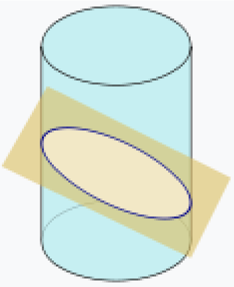
Right circular cylinders[edit]

The bare term *cylinder* often refers to a solid cylinder with circular ends perpendicular to the axis, that is, a right circular cylinder, as shown in the figure. The cylindrical surface without the ends is called an *open cylinder*. The formulae for the surface area and the volume of a right circular cylinder have been known from early antiquity.

A right circular cylinder can also be thought of as the solid of revolution generated by rotating a rectangle about one of its sides. These cylinders are used in an integration technique (the "disk method") for obtaining volumes of solids of revolution.[3]

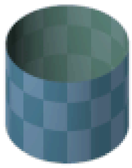
Properties[edit]

Cylindric sections[edit]


Cylindric section

A cylindric section is the intersection of a cylinder's surface with a plane. They are, in general, curves and are special types of *plane sections*. The cylindric section by a plane that contains two elements of a cylinder is a parallelogram.[4] Such a cylindric section of a right cylinder is a rectangle.[4]

A cylindric section in which the intersecting plane intersects and is perpendicular to all the elements of the cylinder is called a *right section*.[5] If a right section of a cylinder is a circle then the cylinder is a circular cylinder. In more generality, if a right section of a cylinder is a conic section (parabola, ellipse, hyperbola) then the solid cylinder is said to be parabolic, elliptic and hyperbolic respectively.



Cylindric sections of a right circular cylinder

For a right circular cylinder, there are several ways in which planes can meet a cylinder. First, planes that intersect a base in at most one point. A plane is tangent to the cylinder if it meets the cylinder in a single element. The right sections are circles and all other planes intersect the cylindrical surface in an ellipse.[6] If a plane intersects a base of the cylinder in exactly two points then the line segment joining these points is part of the cylindric section. If such a plane contains two elements, it has a rectangle as a cylindric section, otherwise the sides of the cylindric section are portions of an ellipse. Finally, if a plane contains more than two points of a base, it contains the entire base and the cylindric section is a circle.

In the case of a right circular cylinder with a cylindric section that is an ellipse, the eccentricity e of the cylindric section and semi-major axis a of the cylindric section depend on the radius of the cylinder r and the angle α between the secant plane and cylinder axis, in the following way:

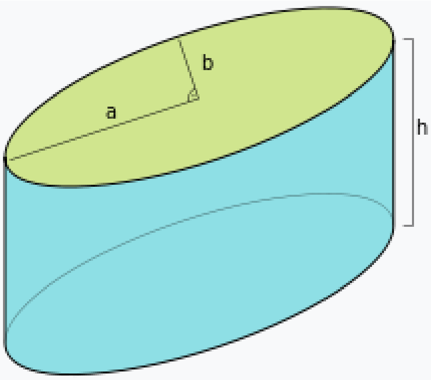
Volume[edit]

If the base of a circular cylinder has a radius r and the cylinder has height h, then its volume is given by

$$V = \pi r^2 h.$$

This formula holds whether or not the cylinder is a right cylinder.[7]

This formula may be established by using Cavalieri's principle.



A solid elliptic cylinder with the semi-axes a and b for the base ellipse and height h

In more generality, by the same principle, the volume of any cylinder is the product of the area of a base and the height. For example, an elliptic cylinder with a base having semi-major axis a, semi-minor axis b and height h has a volume $V = Ah$, where A is the area of the base ellipse (= πab). This result for right elliptic cylinders can also be obtained by integration, where the axis of the cylinder is taken as the positive x-axis and $A(x) = A$ the area of each elliptic cross-section, thus:

Using cylindrical coordinates, the volume of a right circular cylinder can be calculated by integration over

Surface area[edit]

Having radius r and altitude (height) h, the surface area of a right circular cylinder, oriented so that its axis is vertical, consists of three parts:

- the area of the top base: πr^2
- the area of the bottom base: πr^2
- the area of the side: $2\pi rh$

The area of the top and bottom bases is the same, and is called the *base area*, B. The area of the side is known as the *lateral area*, L.

An *open cylinder* does not include either top or bottom elements, and therefore has surface area (lateral area)

$$L = 2\pi rh.$$

The surface area of the solid right circular cylinder is made up the sum of all three components: top, bottom and side. Its surface area is therefore,

$$A = L + 2B = 2\pi rh + 2\pi r^2 = 2\pi r(h + r) = \pi d(r + h),$$

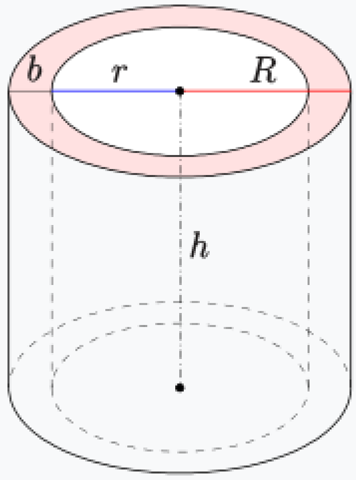
where $d = 2r$ is the diameter of the circular top or bottom.

For a given volume, the right circular cylinder with the smallest surface area has $h = 2r$. Equivalently, for a given surface area, the right circular cylinder with the largest volume has $h = 2r$, that is, the cylinder fits snugly in a cube of side length = altitude (= diameter of base circle).[8]

The lateral area, L, of a circular cylinder, which need not be a right cylinder, is more generally given by:

$$L = e \times p,$$

where e is the length of an element and p is the perimeter of a right section of the cylinder.[9] This produces the previous formula for lateral area when the cylinder is a right circular cylinder.



Hollow cylinder

Right circular hollow cylinder (cylindrical shell)[edit]

A *right circular hollow cylinder* (or *cylindrical shell*) is a three-dimensional region bounded by two right circular cylinders having the same axis and two parallel annular bases perpendicular to the cylinders' common axis, as in the diagram.

Let the height be h, internal radius r, and external radius R. The volume is given by

.

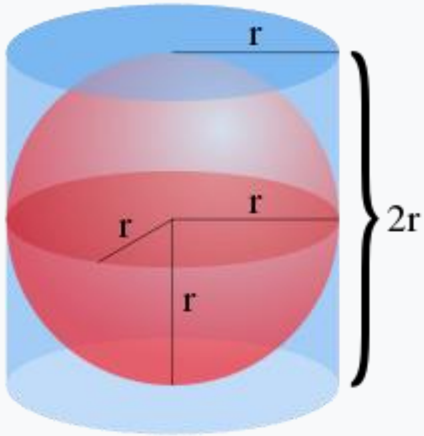
Thus, the volume of a cylindrical shell equals 2π(average radius)(altitude)(thickness).[10]

The surface area, including the top and bottom, is given by

.

Cylindrical shells are used in a common integration technique for finding volumes of solids of revolution.[11]

On the Sphere and Cylinder[edit]



A sphere has 2/3 the volume and surface area of its circumscribing cylinder including its bases

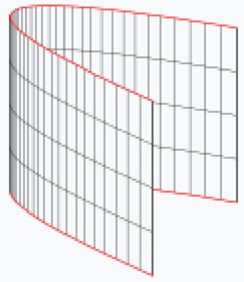
Main article: On the Sphere and Cylinder

In the treatise by this name, written c. 225 BCE, Archimedes obtained the result of which he was most proud, namely obtaining the formulas for the volume and surface area of a sphere by exploiting the relationship between a sphere and its circumscribed right circular cylinder of the same height and diameter. The sphere has a volume two-thirds that of the circumscribed cylinder and a surface area two-thirds that of the cylinder (including the bases). Since the values for the cylinder were already known, he obtained, for the first time, the corresponding values for the sphere. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3 = \frac{2}{3}(2\pi r^3)$. The surface area of this sphere is $4\pi r^2 = \frac{2}{3}(6\pi r^2)$. A sculpted sphere and cylinder were placed on the tomb of Archimedes at his request.

Cylindrical surfaces[edit]

In some areas of geometry and topology the term *cylinder* refers to what has been called a *cylindrical surface*. A cylinder is defined as a surface consisting of all the points on all the lines which are parallel to a given line and which pass through a fixed plane curve in a plane not parallel to the given line.[12] Such cylinders have, at times, been referred to as *generalized cylinders*. Through each point of a generalized cylinder there passes a unique line that is contained in the cylinder.[13] Thus, this definition may be rephrased to say that a cylinder is any ruled surface spanned by a one-parameter family of parallel lines.

A cylinder having a right section that is an ellipse, parabola, or hyperbola is called an *elliptic cylinder*, *parabolic cylinder* and *hyperbolic cylinder*, respectively. These are degenerate quadric surfaces.[14]



Parabolic cylinder

When the principal axes of a quadric are aligned with the reference frame (always possible for a quadric), a general equation of the quadric in three dimensions is given by

with the coefficients being real numbers and not all of A, B and C being 0. If at least one variable does not appear in the equation, then the quadric is degenerate. If one variable is missing, we may assume by an appropriate rotation of axes that the variable z does not appear and the general equation of this type of degenerate quadric can be written as[15]

where

If $AB > 0$ this is the equation of an *elliptic cylinder*.[15] Further simplification can be obtained by translation of axes and scalar multiplication. If has the same sign as the coefficients A and B, then the equation of an elliptic cylinder may be rewritten in Cartesian coordinates as:

This equation of an elliptic cylinder is a generalization of the equation of the ordinary, *circular cylinder* ($a = b$). Elliptic cylinders are also known as *cylindroids*, but that name is ambiguous, as it can also refer to the Plücker conoid.

If has a different sign than the coefficients, we obtain the *imaginary elliptic cylinders*:

which have no real points on

them. (gives a single real point.)

If A and B have different signs

and , we obtain the *hyperbolic cylinders*, whose equations may be rewritten as:

Finally, if $AB = 0$ assume, without loss of generality, that $B = 0$ and $A = 1$ to obtain the *parabolic cylinders* with equations that can be written as:[16]

Conclusion:

The conclusion highlights the significance of right circular cylinders in both theoretical geometry and practical applications, emphasizing their geometric properties, formulas for volume and surface area, and their widespread use across various fields.

A Field Project Report

on

SPHERE THROUGH A GIVEN CIRCLE

Submitted by

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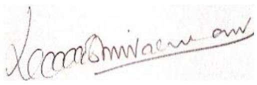
(ACCREDITED BY "NAAC A+" GRADE)

VADLAMUDI, GUNTUR – 52213, AP, INDIA

APRIL – 2023

CERTIFICATE

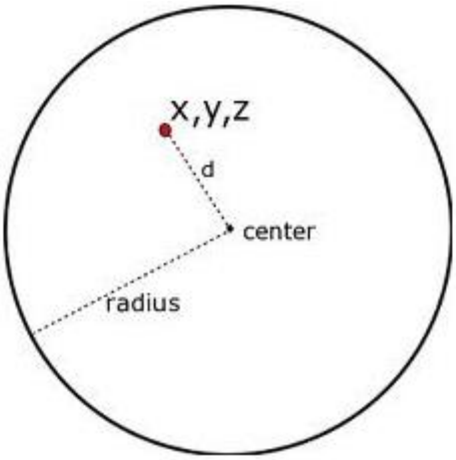
This is to certify that the project report titled “**SPHERE THROUGH A GIVEN CIRCLE”** Submitted by **SHAIK NURUDDIN BASHA (211FM01051), PATHA SAIKUMAR ((211FM01053)), KAITHEPALLI MRUDULA RAJESWARI (211FM01053)** is carried out as field project work under my supervision. I approve this field project work for submission towards partial fulfilment of the requirements and course work in prescribed for B.Sc., VFSTR (Deemed to be University).



Project Guide   Head of Department

INTRODUCTION:

A sphere is defined as a completely round geometrical object in a three-dimensional space just like a round ball. To be geometrical, a sphere is a set of points that are equidistant from a point in space. The distance between the outer point and centre of the sphere is called the radius, denoted by r and the maximum straight distance between any two sides of the sphere through the centre is known as the diameter, denoted by d.



A hemisphere is exactly half of a sphere which can only be obtained when a sphere is split from the middle. The biggest circle of a sphere is a circle that has the same centre and radius of a sphere. A great circle of the sphere is a circle that has the same radius and centre as the sphere itself.

Abstract:

A circle is a closed figure that can be drawn using a constant length from a fixed-point center. A sphere is a three-dimensional circle. Circle and sphere both is round and measured using radius. In this study with the help of the concepts of the sphere and circle we can get a sphere equation through a given circle.

What is the Sphere?

A sphere is a three-dimensional counterpart of a circle, with all its points lying in space at a constant distance from the fixed point or the center, called the radius of the sphere. Radius of the sphere denoted as r.

The general **equation** of a **sphere** is: $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$, where (a, b, c) represents the center of the **sphere**, r represents the radius, and x, y, and z are the coordinates of the points on the surface of the **sphere**.

What is a Circle?
A circle is a round shaped figure that has no corners or edges.

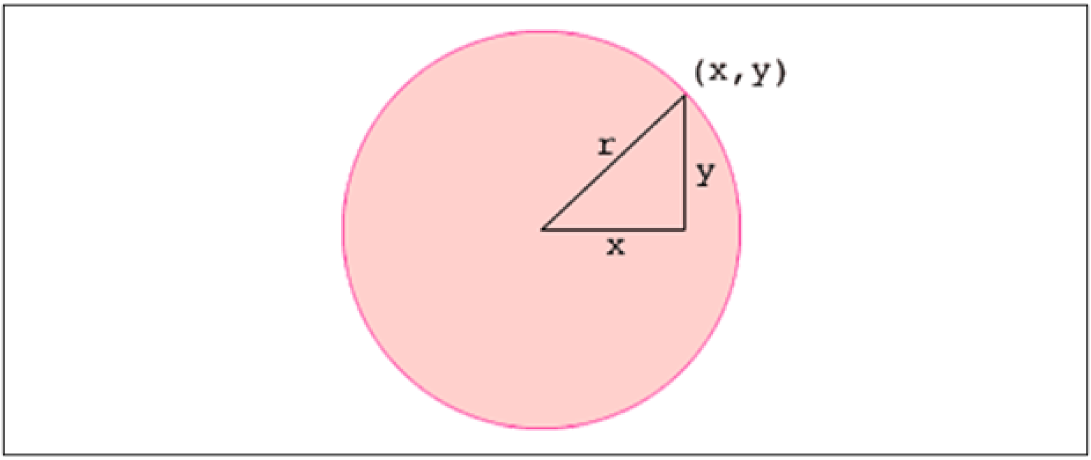
In geometry, a circle can be defined as a closed, two-dimensional curved shape.
The General Form of the equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$.
The centre of the circle is (-g, -f) and the radius is $\sqrt{(g^2 + f^2 - c)}$.

How to Derive the Equation of a Sphere?

The equation of a circle of radius r is given by:

$x^2 + y^2 = r^2$

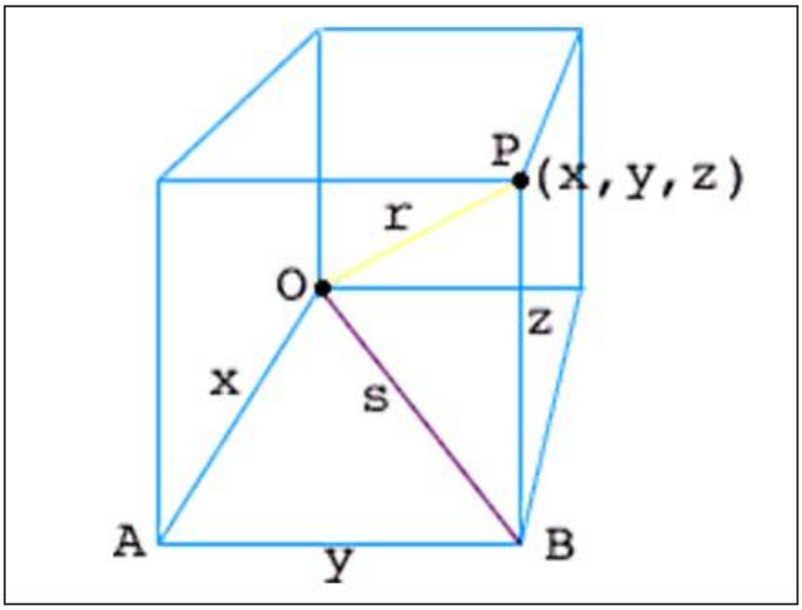
You can relate it to the algebraic method of starting the Pythagoras theorem.



The point (x,y) lies on the circle only when the right triangle has sides of length |x| and |y| and hypotenuse of length r, which can be written as:

$x^2 + y^2 = r^2$

Pythagoras theorem can be used twice for the equation of a sphere. In the below figure, O is the origin and P(x,y,z) is a point in three-space. P is on the sphere with radius r only when the distance from O to P is r.



Since OAB is a right angle triangle, $x^2 + y^2 = s^2$. The triangle OBP is another right triangle and therefore, $s^2 + z^2 = r^2$. Hence, the distance between O and P can be expressed by:

$x^2 + y^2 + z^2 = |OP|^2$

Hence, we can conclude that (x,y,z) lies on the sphere with radius r only if,

$$\mathbf{x^2 + y^2 + z^2 = r^2}$$

which is called the equation of a sphere.

If (a, b, c) is the centre of the sphere, r represents the radius, and x, y, and z are the coordinates of the points on the surface of the sphere, then the general equation of a sphere is $(x – a)^2 + (y – b)^2 + (z – c)^2 = r^2$

Example Problem

1. Find the equation of the sphere which passed through the point (1, -2, 3) and the circle

   $z=0,\ x^2+y^2+z^2 -9 = 0$

Solution:

Any sphere through the given circle is $(x^2+y^2+z^2 -9) + \lambda z =0$ -----------(1)

If if passes through the given point (1, -2, 3) we get,

$(1+4+9-9) + \lambda(3) = 0$

$3\lambda = -5$

$\lambda = -5/3$ ---------------------(2)

Substitute equation (2) in equation (1) we get,

$(x^2+y^2+z^2 -9) -5/3\ z =0$

$3(x^2+y^2+z^2) – 5z - 27=0$, which is the required equation of the sphere.

Conclusion:

It sounds like you're referring to the problem of finding a sphere that passes through a given circle in three-dimensional space. The conclusion would depend on the approach and solution method used to solve the problem.

A Field Project Report

on

Properties of Matrix Group (G) under Multiplication

Submitted by

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CERTIFICATE

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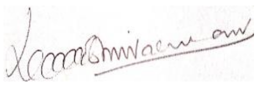

Project Guide

Head of Department

Abstract: This paper explores the fundamental properties of a matrix group denoted under the operation of matrix multiplication. The properties discussed include closure, associativity, the existence of an identity matrix, the existence of inverses for non-singular matrices, and the non-abelian property of the group. Through rigorous examination, it is demonstrated that satisfies the closure property, and associativity property, and possesses an identity matrix. Furthermore, every non-singular matrix has an inverse. Finally, a counterexample illustrates the non-abelian property of the matrix group, emphasizing the importance of understanding these properties in the realm of matrix algebra and group theory.

Keywords: Matrix Group, Matrix Multiplication, Closure, Associativity, Identity Matrix, Inverses, Non-singular Matrices, Non-abelian Property

Introduction

Matrix groups play a pivotal role in various branches of mathematics and sciences, particularly in linear algebra, group theory, and applications ranging from physics to computer graphics. Understanding the properties of these matrix groups under multiplication is essential for analyzing their behavior and applications. In this introduction, we embark on a journey to explore the fundamental properties exhibited by a matrix group denoted as when subjected to the operation of matrix multiplication.

The properties to be investigated include closure, associativity, the existence of an identity matrix, the presence of inverses for non-singular matrices, and the non-abelian nature of the group. These properties form the cornerstone of group theory and provide valuable insights into the structure and behavior of matrix groups.

Through a systematic examination of each property, we aim to establish a comprehensive understanding of the behavior of matrix groups under multiplication. By elucidating these properties, we lay the groundwork for further exploration into advanced topics in linear algebra, group theory, and their diverse applications.

Join us as we delve into the intricate world of matrix groups and unravel the fascinating properties that govern their behavior under the operation of multiplication.

Background Study

The study of matrix groups and their properties under multiplication lies at the intersection of linear algebra and group theory, two fundamental branches of mathematics with broad applications across various disciplines. Understanding these properties requires a foundation in both areas, as outlined below

Linear Algebra

Matrix groups are deeply rooted in linear algebra, where matrices are used to represent linear transformations and solve systems of linear equations. A solid understanding of concepts such as matrix operations (addition, multiplication), determinants, inverses, and eigenvalues/eigenvectors is essential for comprehending the properties of matrix groups under multiplication. Linear algebra provides the necessary tools to analyze the behavior of matrices and their interactions within groups.

Group Theory

Group theory is the branch of mathematics concerned with the study of groups, which are sets equipped with a binary operation satisfying certain axioms. Matrix groups form a specific class of groups where the elements are matrices, and the binary operation is matrix multiplication. Knowledge of group axioms, subgroup structures, co-sets, and group homeomorphisms is fundamental for understanding the properties exhibited by matrix groups under multiplication.

Properties of Groups

To study matrix groups under multiplication, one must be familiar with the fundamental properties of groups, such as closure, associativity, identity elements, and inverses. These properties provide a framework for analyzing the behavior of matrix groups and verifying their characteristics. Additionally, understanding the concept of non-abelian groups is crucial for recognizing when matrix groups do not commute under multiplication.

Applications

Beyond theoretical considerations, the properties of matrix groups under multiplication have numerous practical applications in various fields. For instance, in physics, matrix groups are used to represent symmetries and transformations in quantum mechanics and special relativity. In computer graphics, they are employed to manipulate images and perform geometric transformations. Familiarity with the properties of matrix groups enables researchers and practitioners to leverage their power in solving real-world problems.

Overall, a comprehensive background study encompassing linear algebra, group theory, properties of groups, and practical applications forms the basis for delving into the properties of matrix groups under multiplication. By synthesizing knowledge from these areas, researchers can gain insights into the behavior and applications of matrix groups in diverse mathematical and scientific contexts.

System Tool

What is the matrix

A matrix is a fundamental mathematical concept used across various fields like linear algebra, physics, engineering, computer science, and economics. It consists of a rectangular array of numbers, symbols, or expressions arranged in rows and columns. Each element in a matrix is located at a specific intersection of a row and column, identifiable by its row and column indices. Matrices play a crucial role in representing systems of linear equations, geometric transformations, data organization, and modeling scientific and engineering problems. Operations such as addition, subtraction, scalar multiplication, and matrix multiplication are defined and analyzed using matrices. The size of a matrix, determined by its number of rows and columns, is denoted by m * n for an m * n matrix, where a-ij represents the element at the i-th row and j -th column.

Applications of matrix Group under Multiplication in daily life

Matrix groups under multiplication find numerous applications in daily life across various fields. Here are some examples:

Computer Graphics

In computer graphics, matrices are used to represent transformations such as translation, rotation, scaling, and shearing. These transformations are applied to graphical objects to simulate movements, changes in size, and orientations in video games, animations, virtual reality, and computer-aided design (CAD) software.

Finance

Matrix multiplication is utilized in finance for portfolio management, risk assessment, and investment strategies. Matrices can represent historical price data, portfolio weights, and correlations between assets. Multiplying matrices enables analysts to compute returns, calculate risk measures, and optimize investment portfolios efficiently.

Engineering

Engineers use matrix groups to solve systems of linear equations arising in various engineering problems, such as structural analysis, circuit design, signal processing, and control systems. Matrices represent physical quantities, such as forces, voltages, and signals, and matrix operations help analyze and manipulate these quantities to design and optimize engineering systems.

Transportation and Logistics

Matrix multiplication plays a vital role in transportation and logistics for route optimization, scheduling, and resource allocation. Matrices can represent factors like distances between locations, traffic flows, and transportation costs. By multiplying matrices, logistics companies can determine the most efficient routes, minimize travel times, and allocate resources effectively.

Image Processing

In image processing, matrices are used to represent digital images, where each element of the matrix corresponds to the intensity of a pixel. Matrix operations, including multiplication, are applied to perform tasks such as image filtering, convolution, edge detection, and image enhancement. These techniques are widely used in photography, medical imaging, and video processing applications.

Artificial Intelligence and Machine Learning

Matrix operations are fundamental to many algorithms in artificial intelligence and machine learning, including neural networks, deep learning, and data analysis. Matrices represent input data, model parameters, and intermediate computations in these algorithms. Multiplying matrices enables the training and inference processes, where models learn from data and make predictions or classifications.

Cryptographic Systems

Matrix multiplication is used in cryptographic systems for encryption and decryption processes. Matrices serve as encryption keys and are multiplied with plaintext messages to produce cipher text. By using appropriate matrix operations, cryptographic algorithms ensure data confidentiality and security in communication systems, financial transactions, and digital signatures.

These are just a few examples illustrating the wide-ranging applications of matrix groups under multiplication in everyday life. From computer graphics to finance, engineering, transportation, image processing, artificial intelligence, and cryptography, matrices play a crucial role in solving complex problems and advancing technology and innovation.

easy of use

Understanding the properties of matrix groups under multiplication offers several advantages in terms of ease of use

Intuitive Concepts

The basic properties such as closure, associativity, and identity are easy to understand intuitively. Closure means that multiplying any two matrices within the group will still yield a matrix within the group. Associativity ensures that the order of operations doesn't affect the result, much like simple arithmetic operations. The concept of an identity matrix acting as a neutral element is similar to the role of zero in addition.

Clear Notation

Matrices are typically represented using straightforward notation, with elements arranged in rows and columns. Each element is identified by its row and column indices, making it easy to identify and manipulate elements within the matrix.

Practical Applications

Understanding these properties provides practical benefits in various fields. For instance, in computer graphics, knowing how matrices behave under multiplication allows for efficient transformations of graphical objects. Similarly, in engineering, these properties facilitate the solution of systems of linear equations and the analysis of complex systems.

Computational Tools

With the availability of computational tools and software packages like MATLAB, Python with Numbly, or Wolfram Mathematica, performing matrix operations and verifying properties has become more accessible than ever. These tools offer simple functions and commands to perform matrix operations, making it easier for users to explore and apply matrix properties.

Concrete Examples

Learning through concrete examples helps reinforce understanding. By working through examples in various contexts, such as solving systems of linear equations or analyzing transformations in computer graphics, learners can gain practical experience and confidence in applying matrix properties.

Algorithmic Verification

Verifying properties such as closure or associativity often involves straightforward algorithms or procedures. For example, checking closure requires multiplying matrices and ensuring the result satisfies the group's criteria, a task easily automated through programming or computational tools.

In summary, the properties of matrix groups under multiplication offer intuitive concepts, clear notation, practical applications, computational tools, concrete examples, and algorithmic verification methods, all of which contribute to their ease of use for learners and practitioners alike.

Different Methods To Solve Properties of Matrix Group G under Multiplication

There are several methods to solve for properties of matrix groups under multiplication, each tailored to address specific aspects of the properties being investigated. Here are some common methods:

Direct Verification

One straightforward method is to directly verify the properties by applying the definitions and rules of matrix multiplication. For example, to demonstrate closure, you would multiply two arbitrary matrices from the group and check if the result is also within the group.

Theoretical Proof

Another method involves providing a theoretical proof based on the properties of matrices and group theory. This approach often utilizes mathematical reasoning, logic, and theorems to demonstrate why a certain property holds for the matrix group under multiplication.

Example-based Approach

Using concrete examples can be an effective method to illustrate properties. By selecting specific matrices and demonstrating how the property holds for those examples, learners can gain intuition and understanding of the property's application in practice.

Algorithmic Approach

For certain properties, especially closure and associativity, algorithmic approaches can be used to verify the properties computationally. Algorithms can be written to perform matrix multiplication and check if the resulting matrices satisfy the desired property.

Counterexamples

In some cases, disproving a property using counterexamples can be insightful. By providing specific matrices for which the property does not hold, one can demonstrate the limitations or exceptions to the property.

Geometric Interpretation

For properties related to transformations, a geometric interpretation can be helpful. Visualizing matrices as transformation operators and observing how they interact with geometric shapes or vectors can provide insights into properties such as associativity and closure.

Mathematical Software

Utilizing mathematical software packages like MATLAB, Python with Numbly, or Wolfram Mathematica can expedite the process of solving for properties. These tools offer built-in functions and commands to perform matrix operations and verify properties efficiently.

Theoretical Frameworks

Leveraging established theoretical frameworks from linear algebra and group theory can provide a systematic approach to solving properties of matrix groups. By applying fundamental principles and theorems, one can rigorously analyze and prove properties of interest.

Each of these methods offers unique advantages and may be suitable depending on the specific property being investigated, the complexity of the problem, and the preferences of the researcher or learner. Combining multiple methods can provide a comprehensive understanding of the properties of matrix groups under multiplication.

Objective

a) Demonstrate that if (G, *) is an abelian group, then for all $a, b \in G$ and $\forall\ n \in Z$, $(a*b)$-1= a-1*b-1

b) Let G be the set of all 2x2 matrices $[a\ b\ c\ d]$, where a,b,c,d ∈ R such that ad-bc ≠ 0. Discuss G is a non-abelian group for the multiplication of matrices defined as $[a\ b\ c\ d]$ $[w\ x\ y\ z]$ = $[a$+w b+ x c+ y d+z].

c) Discuss whether the set Q-{1}, rational numbers other than 1 is an abelian group under the composition * defined as x*y=x-y+xy

procedure:

Given (G,*) is abelian

To prove(a b)-1=a-1 b-1

Proof: let "g" be a group and a b are the elements (a b) E g,

Let a-1, and b-1 be the inverse elements

(b-1, a-1) E G

Let c= b-1a-1

According to inverse law

aa-1=a-1a=e

(a b) c =(a b)(b-1 a-1)

= a(bb-1)a-1

=a(ea-1)

= aa-1

Abc = e

Now

C(ab)=(b-1 a-1)ab

= b-1(a-1a)b

= b-1(e)b

= b-1(eb)

= b-1 b

=e

From 1&2

(ab)c= c(ab)

E= (ab)-1

b-1 a-1=(ab)-1

(ab)-1 = b-1.a-1

Hence proved

Given two matrices

A=[a b c d], B= [w x y z]

A+B= [a+w b+x c+y d+z]

Given ad-bc does not equal 0 and wxyz are real numbers

Det(a+b) = (a+w)(d+z)-(b+x)(c+y)

= ad+az+dw+zw-(bc+by+xc+xy)

= ad+az+dw+zw-bc-by-xc-xy

The determinant is not equal to zero

A+B is in G is Closed under matrix addition

Associative:

Matrix addition is known to be associative, so this property holds for the matrices in G.

Identity:

The identity of 2*2 matrices is matrix i= [1 0 0 1]

The determinant of I is 1

A+I=[a b c d]+[1 0 0 1]

= [a b c d]

= A

I+A = [1 0 0 1]+[a b c d]

= [a b c d]

= A

A+I=I+A=A, So ‘I’ serves as the identity element for G

Inverse:

Let's take A= [a b c d]

A-1= 1/ ad-bc[+d –d –c a]

= 1/ad-bc[ad-bc]

= 1

So, A-1 is also in G.

Communicative:

Consider,

A=[a b c d],B= [w x y z]

A+B = [a+w b+x c+y d+z]

Similarly

B+A=[w+a x+b y+c z+d]

A+B=B+A

A+B is in G is Commutative under all 2*2 matrices.

Given X, Y E Q-{1}

Closure property:
Let x,y E q-{1}

Now, x*y= x-y+xy E q-{1}

Hence, q-{1} satisfies closure property under ‘x’

Associative property:
Let x,y,z Eq –{1}

Now,

(x*y)*z

LHS=(x-y+xy)*z

LHS= [(x-y+xy)z+(x-y+xy)z]

= x-y+xy-z+xz-yz+xyz

RHS= x*(y*z)

= x=(y-z+yz)

= [x-(y-z+yz)+x(y-z+yz)]

=x-y+z-yz+xy-xy-xz+xyz

LHS Not Equal RHS

Therefore Q-{1} is not satisfy associative property under’*’.

G is a not-abelian group under ‘*’ defined by x*y=x-y+xy.

Acknowledgment

We would like to express our sincere gratitude to all those who have contributed to the exploration and understanding of the properties of matrix groups under multiplication. First and foremost, we extend our appreciation to the pioneers and researchers in the fields of linear algebra, group theory, and mathematics, whose groundbreaking work has laid the foundation for our understanding of matrices and their properties. We are deeply thankful to our educators, mentors, and academic advisors, whose guidance and support have been invaluable in shaping our understanding of matrix groups and their properties. Their dedication to teaching and mentorship has inspired us to delve deeper into this fascinating area of mathematics.

We also acknowledge the creators of mathematical software packages and computational tools that have facilitated our exploration and analysis of matrix groups. These tools have provided us with the means to perform complex calculations, verify properties, and visualize mathematical concepts effectively. Furthermore, we would like to express our gratitude to our peers and colleagues with whom we have engaged in discussions, collaborations, and knowledge-sharing activities. Their insights, feedback, and collaboration have enriched our understanding of matrix groups and enhanced the quality of our work. Last but not least, we extend our heartfelt thanks to our families, whose unwavering support and encouragement have been instrumental in our academic and professional pursuits.

This acknowledgment is a testament to the collective effort and collaboration that have contributed to our exploration of the properties of matrix groups under multiplication. We are deeply grateful for the opportunity to study and learn in this fascinating field of mathematics.

conclusion

The properties of matrix groups under multiplication represent fundamental concepts in mathematics with wide-ranging applications across various fields. Through our exploration, we have gained a deeper understanding of these properties and their significance in linear algebra, group theory, and practical applications.

From closure and associativity to the existence of identity and inverses, these properties define how matrices behave when subjected to multiplication operations. They provide a framework for analyzing the structure and behavior of matrix groups, enabling us to solve systems of linear equations, perform geometric transformations, and model real-world phenomena effectively. Furthermore, the non-abelian property highlights the non-commutative nature of matrix multiplication, underscoring the importance of order in matrix operations. This property has implications in fields such as computer graphics, cryptography, and physics, where the sequence of transformations or operations matters.

In conclusion, the study of properties of matrix groups under multiplication offers a rich and diverse landscape for exploration and application. By mastering these properties, we gain insight into the fundamental principles underlying matrices and their role in mathematics and beyond. As we continue to explore and apply these concepts, we unlock new avenues for innovation, problem-solving, and discovery in the realm of mathematics and its applications.

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MATHEMATICS – FIELD PROJECT REPORT

on

Applications of ODE-CR Circuits

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VADLAMUDI, GUNTUR-522213, AP,INDIA

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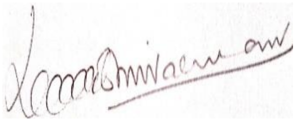
Certificate

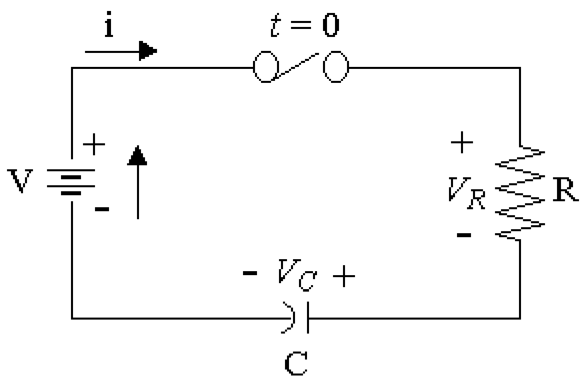
This is to certify the Project report titled **"Applications of ODE - CR Circuits"** submitted by **ALLA VARSHA (201FM01001), DASARI VENKATA SIVA MOHANA AKASH (201FM01002), GADAMSETTY PRUDHVI TEJA (201FM01003), JONNAKUTI SATHU DIVYA SRI (201FM01004), PADARTI DURGA LAKSHMI RAGHAVA ABHIGNA (201FM01006)** is carried out as field project work under by supervisor. I approve this field project work for submission towards partial fulfillment of the requirements and course work in prescribed for B. Sc, VFSTR (Deemed to be University)



Project Guide **Head of the Department**

Abstract: This work investigates the application of RLC diagrams in the catena study of linear RLC closed series electric circuits. The Relevant second order ordinary differential equations were solved by Kirchhoff's Voltage law. This solution obtained was employed to procedure RLC diagram simulated by MATLAB and Mathematica 9.0. A circuit containing an inductance L or capacitor C and resistor R with current and voltage variable given by differential equation. The general solution of differential equation has two parts complementary function (C. F) and particular integral (P. I) in which C. F. represents transient response and P. I. represents steady response. The general solution of differential equation represents the complete response of network. In this connection, this paper includes RLC circuit and ordinary differential equation of second order and its solution.

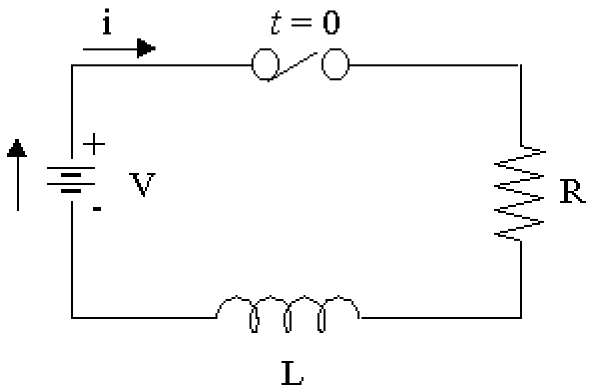
Introduction: Series RC Circuit



An RC series circuit

In this section we see how to solve the differential equation arising from a circuit consisting of a resistor and a capacitor. In an RC circuit, the **capacitor** stores energy between a pair of plates. When voltage is applied to the capacitor, the charge builds up in the capacitor and the current drops off to zero.

Description: Formation of Ordinary Differential Equations: Series RL Circuit



RL circuit diagram

The RL circuit shown above has a resistor and an inductor connected in series. A constant voltage *V* is applied when the switch is closed.

Case 1: Constant Voltage

The voltage across the resistor and capacitor are as follows:

$$V_R = Ri$$

and

$$V_C = \frac{1}{C}\int i\,dt$$

Kirchhoff's voltage law says the total voltages must be zero. So applying this law to a series RC circuit results in the equation:

$$Ri + \frac{1}{C}\int i\,dt = V$$

One way to solve this equation is to turn it into a **differential equation**, by differentiating throughout with respect to t:

$$R\frac{di}{dt} + \frac{i}{C} = 0$$

Solving the equation gives us:

$$i = \frac{V}{R}e^{-t/RC}$$

We start with:

$$R\frac{di}{dt} + \frac{i}{C} = 0$$

Divide through by R:

$$\frac{di}{dt} + \left(\frac{1}{RC}\right)i = 0$$

We recognise this as a first order linear differential equation.

Identify P and Q:

$$P = \frac{1}{RC}$$

Q = 0

Find the integrating factor (our independent variable is t and the dependent variable is i):

$$\int P\,dt = \int \frac{1}{RC}\,dt = \frac{1}{RC}t$$

So

$$IF = e^{t/RC}$$

Now for the right hand integral of the 1st order linear solution:

$$\int Qe^{\int P\,dt}\,dt = \int 0\,dt = K$$

Applying the linear first order formula:

$$ie^{t/RC} = K$$

Since $i = \frac{V}{R}$ when $t = 0$:

$$K = \frac{V}{R}$$

Substituting this back in:

$$ie^{t/RC} = \frac{V}{R}$$

Solving for i gives us the required expression:

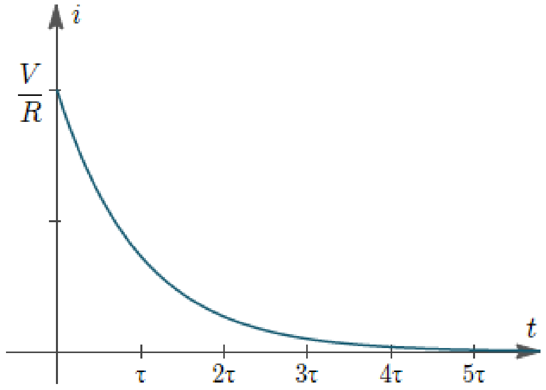
Solving for i gives us the required expression:

$$i = \frac{V}{R}e^{-t/RC}$$

The function

$$i = \frac{V}{R}e^{-t/RC}$$

has an **exponential decay** shape as shown in the graph. The current stops flowing as the capacitor becomes fully charged.



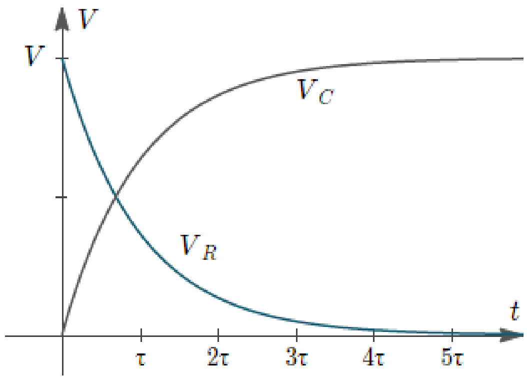
Graph of $i = \frac{V}{R}e^{-(t/RC)}$, an exponential decay curve.

Applying our expressions from above, we have the following expressions for the voltage across the resistor and the capacitor:

$$V_R = Ri = Ve^{-t/RC}$$

$$V_C = \frac{1}{C}\int i\,dt = V\left(1 - e^{-t/RC}\right)$$

While the voltage over the resistor drops, the voltage over the capacitor rises as it is charged:



Graphs of $V_R = Ve^{-t/RC}$ (in green) and $V_C = V\left(1 - e^{-t/RC}\right)$ (in gray).

Case 2: Variable Voltage and 2-mesh Circuits

We need to solve variable voltage cases in q, rather than in i, since we have an integral to deal with if we use i.

So we will make the substitutions:

$$i = \frac{dq}{dt}$$

and

$$q = \int i\,dt$$

and so the equation in i involving an integral:

$$Ri + \frac{1}{C}\int i\,dt = V$$

becomes the differential equation in q:

$$R\frac{dq}{dt} + \frac{1}{C}q = V$$

PROBLEM:

Find the charge and the current for $t > 0$ in a series RC circuit where R = 10 W, $C = 4 \times 10^{-3}$ F and $E = 85 \cos 150t$ V.

Assume that when the switch is closed at $t = 0$, the charge on the capacitor is -0.05 C.

SOLUTION:

We will solve this 2 ways:

1. Solving in q.

2. Using Scientific Notebook.

[**NOTE:** We cannot use the formulae $V_C = V\left(1 - e^{-t/RC}\right)$ and $i = \frac{V}{r}e^{-t/RC}$, since the voltage source is **not constant**.]

From the formula: $Ri + \frac{1}{C}\int i\,dt = V$, we obtain:

$$R\frac{dq}{dt} + \frac{1}{C}q = V$$

Since $R = 10$, $C = 4 \times 10^{-3}$, and $V = 85\cos 150t$, we have:

$$10\frac{dq}{dt} + \frac{1}{4 \times 10^{-3}}q = 85\cos 150t$$

$$10\frac{dq}{dt} + 250q = 85\cos 150t$$

$$\frac{dq}{dt} + 25q = 8.5\cos 150t$$

Now, we can solve this differential equation in q using the **linear DE** process as follows:

$$\text{IF} = e^{25t}$$

$$e^{25t}q = \int e^{25t}8.5\cos 150t\,dt = 8.5\int e^{25t}\cos 150t\,dt$$

Then we use the integration formula (found in our standard integral table):

$$\int e^{mt}\cos nt\,dt = \frac{e^{mt}}{m^2 + n^2}(m\cos nt + n\sin nt)$$

We obtain:

$$e^{25t}q = 8.5\int e^{25t}\cos 150t\,dt$$

$$= 8.5\frac{e^{25t}}{23125}(25\cos 150t + 150e^{25t}\sin 150t)$$

$$= 0.0092e^{25t}\cos 150t + 0.055e^{25t}\sin 150t + K$$

Dividing throughout by e^{25t} gives:

$$q = 0.0092\cos 150t + 0.055\sin 150t + Ke^{-25t}$$

We now need to find K:

$q(0) = -0.05$ means $K = -0.05 - 0.0092 = -0.0592$

So this gives us:

$q = 0.0092 \cos 150t + 0.055 \sin 150t - 0.0592e^{-25t}$

Method Using Scientific Notebook

We set up the differential equation and the initial conditions in a **matrix** (not a table) as follows:

$$\frac{dq}{dt} + 25q = 8.5 \cos 150t$$

$$q(0) = -0.05$$

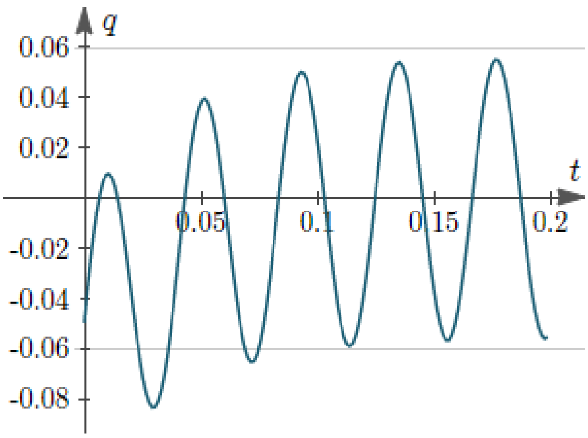
Choosing **Solve ODE - Exact** from the **Compute** menu gives:

Exact solution is:

$q(t) = 0.0092 \cos 150t + 0.055 \sin 150t - 0.059e^{-25t}$

The graph for $q(t)$ is as follows:

The graph for $q(t)$ is as follows:

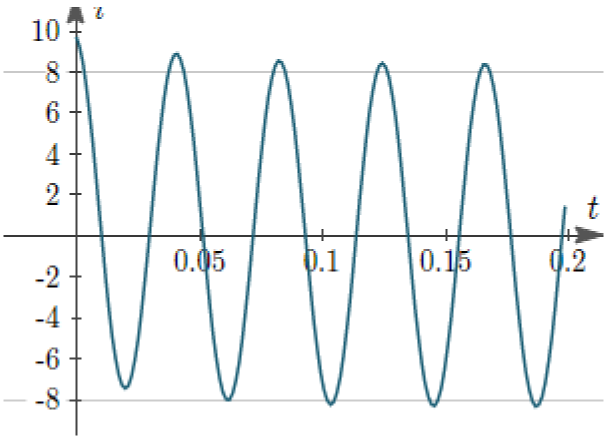


Graph of current q at time t. It's in steady state by around $t = 0.12$.

We are also asked to find the current. We simply differentiate the expression for q:

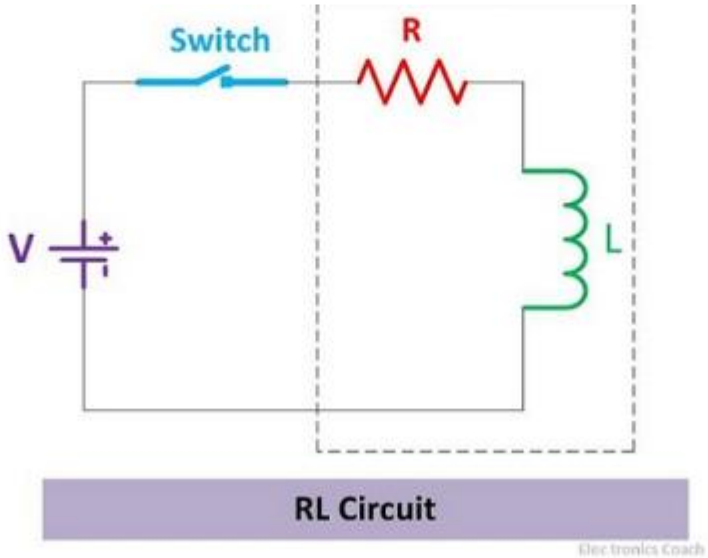
$$i = \frac{d}{dt}(0.0092 \cos 150t + 0.055 \sin 150t - 0.0592e^{-25t}) = -1.38 \sin 150t + 8.25 \cos 150t + 1.48e^{-25t}$$

The graph for $i(t)$:



Graph of current i at time t. It's also in steady state by around $t = 0.12$.

Limitations of LR Circuits



1. The RC and RL circuit, both stores energy, but the RC circuit stores energy in the form of an electric field. While RL circuit stores energy in the form of magnetic field.

2. The RC circuits are economical as capacitors are cheap and abundantly available while inductors are costly which makes RL circuit expensive.

3. The inductors possess a wider tolerance ratings in comparison with resistors, thus RL circuit has high tolerance values.

4. The inductor generates the magnetic field which creates noise in the circuit. This leads to poor performance of RL circuit when

the noise signal becomes high. The problem of noise can be mitigated by using RC circuit as the capacitor does not generate the magnetic field.

Conclusion: In conclusion, our project delves into the application of Ordinary Differential Equations (ODEs) within the framework of RC and RL circuits, illuminating their critical roles in electrical engineering. By employing Kirchhoff's Voltage Law and solving relevant second-order ODEs, we effectively demonstrated how these circuits behave over time, differentiating between transient and steady-state responses. Tools like MATLAB and Mathematica 9.0 were instrumental in simulating RLC diagrams, providing a visual comprehension of the theoretical concepts. Despite their utility, we identified inherent limitations within RC and RL circuits, such as energy storage methods and cost implications, alongside tolerance and noise issues, particularly in RL circuits. These findings underscore the importance of selecting appropriate circuit types based on specific application requirements, balancing between efficiency and performance to mitigate potential drawbacks. This project not only enhances our understanding of electrical circuits but also showcases the practical applications of mathematical theories in real-world scenarios.

MATHEMATICS – FIELD PROJECT REPORT

on

Applications of ODE-Freely Falling body

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VADLAMUDI, GUNTUR-522213, AP,INDIA

March,2023

<u>Certificate</u>

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Project Guide **Head of the Department**

Abstract: The motion of a free falling object in kinematic equation is studied using its characteristics and strategy. The paper is based on the definition of Newton's law of motion. By appropriate examples, it is shown how to solve a free falling object with the given parameters in kinematic equation. Some sufficient conditions to determine the motion of a free falling object, based on the four kinematic equations are established. Some examples are illustrated to solve the problems.

Introduction:Excerpts from Newton's Principia

Mass

The quantity of matter is the measure of the same arising from it's density and bulk conjointly.

Motion

The quantity of motion is the measure of the same arising from the velocity and quantity of matter conjointly.

Force

The *vis insita:* an innate forces of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forward in a right line."

Force

An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.

Law 1

Description:The change of motion is proportional to the applied force and takes place in the direction of the straight line along which that force acts.

Laws 2

To every action there is always an equal and contrary reaction; or the mutual actions of any two bodies are always equal and oppositely directed along the same straight line.

Solved Examples

Example 1

Example-1: Compute the height of the body if it has a mass of 2 Kg and touches the ground after 5 seconds?

Solution:

Given parameters are:

Time t = 5 sec

We have to compute the height. So, we can apply the first equation as given above.

i.e. h= 12gt212gt2

Substituting the values,

h= 12gt212gt2
h= 12×9.8×5212×9.8×52

h= 4.9 × 25

h = 122.5 m

Therefore height as required will be 122.5 meter.

Description: An object in motion is characterized by a changing position as a function of time. The derivative of the distance function with respect to time gives us the velocity of the object as a function of time. Furthermore the derivative of the velocity function, gives us the acceleration of the object as a function of time. But what causes an object to move? To understand particle dynamics we need to first understand the concept of force.

Newton's first law of motion states that an object in motion will remain in motion until acted on by a ***force***. This observation is one of most important ones ever made as it offers a way of defining what a force is. According to this law, an object traveling in space at 100,000 km/hr will remain at that velocity forever provided no force acts on it.

For example, if you were to throw a ball in space it would forever continue in the samedirection along with the same velocity with which it left your hand. Here, ***space*** refers to anywhere that is free from the influence of any gravitational force, electro-magnetic force, air-resistance or any other forces. Once set in motion an object will continue with that **same velocity forever**.

Since **no** force is required to keep an object in motion, then a force can be defined as that which **changes** the velocity of the object. Thus force is a measure of a resistance to a change in motion. Since motion is characterized by a constant velocity, then a change in motion results in a change in velocity. By definition, a change in velocity is an **acceleration**.

This simple, yet profound conclusion tells us that forces are defined by accelerations. The force required to accelerate an object is proportional to the magnitude of the acceleration. The mass of the object is also a factor since the greater the mass, the greater its resistance to motion. Observation shows that the resistance to a change in motion is directly dependent on the amount of matter being accelerated.

We can define ***a Newton*** as the force equipped accelerate a body of unit mass, 1 kg, ams2ams2 Therefore to accelerate a body of mass, mm, the required force would be mm times aa.

Force=mass×acceleration(Applications.1)(Applications.1)Force=mass ×acceleration

F(m,a)=ma(Applications.2)(Applications.2)F(m,a)=ma

This is read as, the force required to accelerate a body is directly related to its mass and the magnitude of the acceleration of the mass. The important concept to understand is that forces are defined as accelerations or **changes in velocity**. It requires no force to keep a body in motion Once in motion it will remain in motion. A force is required only to change its velocity or accelerate it. Thus force is a quantifiable measurement of a mass's resistance to a change in motion. If an object had no resistance to a change in motion then there would be no such thing as force!

This might seem to contradict reason. One can better understand this by considering an airplane flying in space, where space is some imaginary place that contains no matter or force fields inside it. If its four engines produce an **acceleration** 5ms25ms2 and the mass of the plane is 8x1068x106 kg, then the thrust or force acting on the plane is (5)·(8x106=4x107)(5)·(8x106=4x107). If we assume an inexhaustible and weightless fuel source then theoretically the engines will push the plane forward with a **constant** force of 4 million newtons.

Now since the plane is flying in an imaginary space under a constant force, it is free to accelerate forever. Remember forces are defined as accelerations and not velocities. The plane will accelerate at a **constant acceleration** of 5ms25ms2. This means the planes **velocity** would increases and increase at the rate of 5mss5mss. The graph of its velocity as a function of time would be a linearly increasing function:

v(t)=5·t(Applications.3)(Applications.3)v(t)=5·t

The derivative of the velocity function is the acceleration function:

a(t)=dvdt=5(Applications.4)(Applications.4)a(t)=dvdt=5

The fundamental concept to understand here is that a force is required only to **change** an objects velocity. A change in velocity is by definition an acceleration. Therefore forces are required only to accelerate an object. A constant force acting on a body will accelerate the body with a constant acceleration, which means the body's velocity will increase and increase forever, all due to a **constant** force. Furthermore the greater the mass, the greater its resistance to a change in velocity. Thus, the force required to accelerate a mass is directly proportional to its mass.

Understanding Free Fall Motion

Having laid down the conceptual basis of what velocity, acceleration and forces are, we can now study the motion of free falling bodies on earth. An object falls to earth because of the gravitational force of attraction that the earth experiences for the object. What then is gravity? From Newton's law of gravitation, the force attracting two bodies is given by:

F gravitational=G·m1·m2d2 (G is the gravitational constant) (Applications.5)(Applications.5)Fgravitational=G·m1·m2d2 (G is the gravitational constant)

To derive this, think of a unit mass of 1 kg separated a distance from a larger mass, $\boldsymbol{M}$. The gravitational force is the force of attraction the larger mass expresses for the unit mass and vice-versa. Observation confirms that the gravitational force is proportional to mass the larger body and decreases with the square of the distance separating them. The reason it is distance squared and not just directly related to the distance is because masses are 3-dimensional. In 3-dimensional space, properties are related to the projected areas as opposed to 2-

dimensional geometry that are dependent on the length only. Therefore, the gravitational force between the two masses is:

Fgravitational=c·M·1d2 (c is the some constant) (Applications.6)(Applications.6)Fgravitational=c·M·1d2 (c is the some constant)

If the unit mass was replaced by a mass ***m***, the force of attraction would be ***m*** times the amount it was with the unit mass.

Fgravitational=G·M1·m2d2(Applications.7)(Applications.7)Fgravitational=G·M1·m2d2

GG replaces \(***c******)*** as a gravitational constant determined from experiments. The gravitational force of earth acting on a body of mass m1m1 located ***near the surface of the earth*** is then:

Fgravity=G·mearth·m1d2(Applications.8)(Applications.8)Fgravity=G·mearth·m1d2

Since most of our free falling bodies occur near the surface of the earth, we can take dd, to be the radius of the earth. Substituting the known values in along with the value for GG**,** reduces the equation to:

Fgravity=9.8·m1(Applications.9)(Applications.9)Fgravity=9.8·m1

Since force equal mama, we have:

m1·aa=9.8=9.8·m1(Applications.10)(Applications.11)(Applications.10)m1·a=9.8·m1(Applications.11)a=9.8

This important result tells us that the acceleration of a body of **any** mass is 9.8 meters per second per second near the surface of the earth. The gravitational force acting on a body near the surface of the earth would be its mass, ***m***, times the constant acceleration, 9.8. For those who have a ground to hold them up this does not mean much but for a free-falling body in air its acceleration as it falls toward the earth will be a constant 9.8ms29.8ms2, regardless of its mass. A body of twice the mass will be pulled in by twice the force, but the acceleration ***due to the force of gravity*** remains the same.

This may sound a bit confusing but just remember any body will fall to the earth with a constant acceleration, **independent** of its mass. In terms of particle motion, mass means nothing for a falling body! While the gravitational force increases with mass, the acceleration remains the same.

We can now write the acceleration function for a falling body near the earth's surface as:

$$a(t)=9.8ms2 \text{ (Constant)(Applications.12)(Applications.12)} a(t)=9.8ms 2 \text{ (Constant)}$$

Since acceleration as a function of time is by definition the derivative of the velocity function with respect to time then what we have is the same as:

$$a(t)=dvdt\therefore v'(t)=v'(t)=9.8\text{(Applications.13)(Applications.14)(Applications.13)}a(t)=dvdt=v'(t)\text{(Applications.14)}\therefore v'(t)=9.8$$

The derivative of the velocity function (acceleration) is 9.8. Since we know that the derivative of any function $c\cdot xnc\cdot xn$ is $c\cdot nxn-1c\cdot nxn-1$, then we can easily find the velocity function since we know what its derivative is.

The process of finding a function, given its derivative is known as **anti-differentiation**. In this case 9.8 can also be written as $9.8\cdot t0(t0=1)9.8\cdot t0(t0=1)$. We see that:

$$(n-1)=0\text{(Applications.15)(Applications.15)}(n-1)=0$$

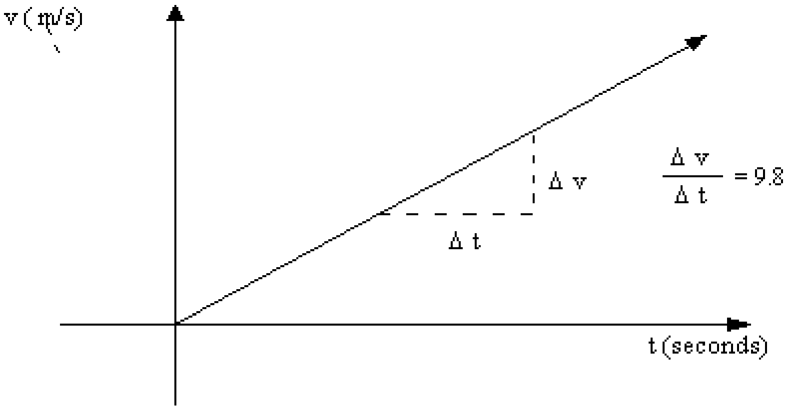
Therefore nn equals 1. Consequently the anti-derivative of $9.8\cdot t09.8\cdot t0$ is $9.8\cdot t9.8\cdot t$. This should hopefully be obvious since the derivative of $v(t)=9.8\cdot tv(t)=9.8\cdot t$ is just $v'(t)=9.8v'(t)=9.8$. We now know that velocity as a function of time is:

$$v(t)=9.9\cdot t\text{(Applications.16)(Applications.16)}v(t)=9.9\cdot t$$

whose derivative with respect to time is:

v′(t)=a(t)=9.8(Applications.17)(Applications.17)v′(t)=a(t)=9.8

The graph of the velocity function is a linearly increasing function with constant rate of change or slope 9.8ms29.8ms2



This graph of the velocity function gives us the objects velocity as any time tt, assuming that air-resistance is negligible. For example at t=10t=10 seconds, the object’s velocity is:

v(t)=9.8·10−98ms(Applications.18)(Applications.18)v(t)=9.8·10−98m s

At t=94t=94 seconds its velocity is:

v(94)=9.8·94=921ms(Applications.19)(Applications.19)v(94)=9.8·94 =921ms

Or almost 30,000kmhr30,000kmhr. Due to air-resistance no object reaches such high velocities. Remember the greater the height it is dropped from the more time it has to increase its velocity or accelerate before it slams into the earth.

Now how do we find the distance function or the distance covered from the point of dropping the object. From the definition of velocity we know that:

v(t)=dsdt∴s′(t)=9.8t=s′(t)(Applications.20)(Applications.21)(Applications.20)v(t)=dsdt=s′(t)(Applications.21)∴s′(t)=9.8t

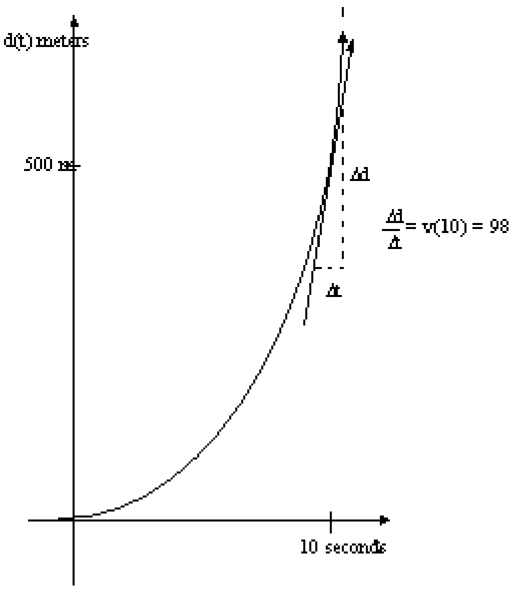
Since s′(t)=9.8·ts′(t)=9.8·t, we have n=1n=1 so its anti-derivative will be 9.8·t229.8·t22, where the derivative of c·t22c·t22 is c·tc· t, ct=9.8tct=9.8t. The solution is then:

d(t)=9.8· t22(or 4.9· t2)(Applications.22)(Applications.22)d(t)=9.8· t22(or 4.9· t2)

The derivative of this function is the velocity function or

v(t)=9.8t(Applications.23)(Applications.23)v(t)=9.8t

We can graph the distance function d(t)=9.8· t22d(t)=9.8· t22. The graph gives us the vertical distance traveled from where it was dropped at any time tt.



Clearly as time, tt, increases, the rate at which distance is being covered is very great . For example between t=0st=0s to t=5st=5s, the object has covered totally.

d(5)−d(t)=9.8· 52−0=122.5 meters(Applications.24)(Applications.24)d(5)−d(t)=9.8· 52−0=122.5 meters

Or the object has covered 112.5 meters in the first five seconds of its free-fall. However from t=20st=20s to t=25st=25s, the object has covered:

d(25)−d(20)=9.8· 2522−9.8· 2022=1102.5 meters(Applications.25)(
Applications.25)d(25)−d(20)=9.8· 2522−9.8· 2022=1102.5 meters

The object has covered more than a kilometer during this five second interval!! This should make sense because initially the body's velocity is small and thus does not cover much distance over a time interval ΔtΔt. However, after some time its velocity has increased (look at the graph of the velocity function graph), such that over a same interval ΔtΔt, the object covers a greater distance. Remember constant acceleration means that the velocity is increasing linearly with time and distance increases with the half square of time.

Conclusion

All objects, irrespective of their mass, experience the same acceleration *g* when falling freely under the influence of gravity at the same point on the Earth. Close to the Earth's surface, g=9.8 m s^{-2}. The weight of an object is the force F_g due to gravity acting on the object, and for an object with mass *m* the weight is given by $F_g=mg$.

If the height of an object of mass *m* changes by Δh, the change in gravitational energy is $\Delta E_g=mg\Delta h$.

If gravity is the only force acting on an object, the sum of kinetic energy and gravitational energy is constant. Increases in kinetic energy are balanced by decreases in gravitational energy, and vice versa.

There are various forms of potential energy, all of which depend on the position of an object rather than on its motion. The potential energy of an object increases as it moves in the opposite direction to that of the force acting on it. Strain energy depends on the extension or compression of an object.

MATHEMATICS – FIELD PROJECT REPORT

on

Applications of ODE-Law of Natural Growth or Decay

Submitted by

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VADLAMUDI, GUNTUR-522213, AP,INDIA

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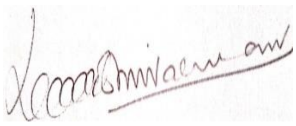
Certificate

This is to certify the Project report titled "**Applications of ODE-Freely Falling body**" submitted by **TOKALA DEEVENA RAJU (201FM01012), TOKALA PRASANTH KUMAR (201FM01013), VEMAVARAPU ADITYA (201FM01014), POKALA HITESH KUMAR (201FM01018), SHAIK MAHABOOB SUBANI (201FM01019)** is carried out as field project work under by supervisor. I approve this field project work for submission towards partial fulfillment of the requirements and course work in prescribed for B. Sc, VFSTR (Deemed to be University)



Project Guide **Head of the Department**

Abstract: One of the most prevalent applications of exponential functions involves growth and decay models. Exponential growth and decay show up in a host of natural applications. From population growth and continuously compounded interest to radioactive decay and Newton's law of cooling, exponential functions are ubiquitous in nature. In this section, we examine exponential growth and decay in the context of some of these applications.

INTRODUCTION:

Exponential Growth Model

Many systems exhibit exponential growth. These systems follow a model of the form $y = y_0 e^{kt}$, where y_0 represents the initial state of the system and k is a positive constant, called the growth constant. Notice that in an exponential growth model, we have

$$y' = ky_0 e^{kt} = ky. \tag{6.8.1}$$

That is, the rate of growth is proportional to the current function value. This is a key feature of exponential growth. Equation 6.8.1 involves derivatives and is called *a differential equation*.

Exponential Growth

Systems that exhibit *exponential growth* increase according to the mathematical model

$$y = y_0 e^{kt} \tag{6.8.2}$$

where y_0 represents the initial state of the system and $k > 0$ is a constant, called the growth constant.

Population growth is a common example of exponential growth. Consider a population of bacteria, for instance. It seems plausible that the rate of population growth would be proportional to the size of the population. After all, the more bacteria there are to reproduce, the faster the population grows. Figure 6.8.1 and Table 6.8.1 represent the growth of a population of bacteria with an initial population of 200 bacteria and a growth constant of 0.02. Notice that after only 2 hours (120 minutes), the population is 10 times its original size!

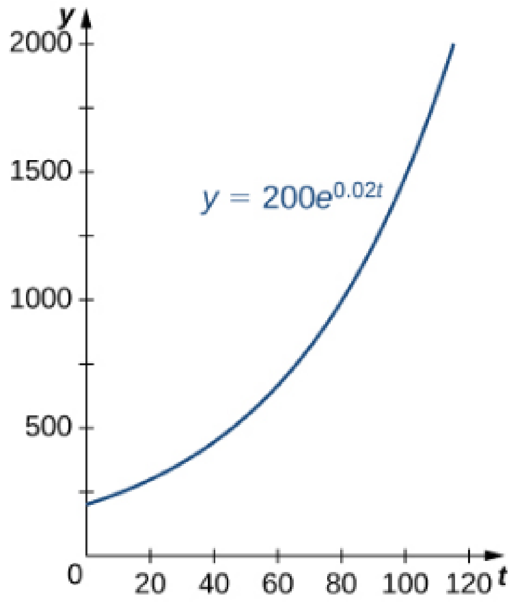


Figure 6.8.1: An example of exponential growth for bacteria.

Table 6.8.1: Exponential Growth of a Bacterial Population

Time(min)	Population Size (no. of bacteria)
10	244
20	298
30	364
40	445
50	544
60	664
70	811
80	991
90	1210
100	1478
110	1805
120	2205

Note that we are using a continuous function to model what is inherently discrete behavior. At any given time, the real-world population contains a whole number of bacteria, although the model takes on noninteger values. When using exponential growth models, we must always be careful to interpret the function values in the context of the phenomenon we are modeling.

Example 1

A herd of llamas has 10001000 llamas in it, and the population is growing exponentially. At time t=4t=4 it has 20002000 llamas. Write a formula for the number of llamas at *arbitrary* time tt.

Solution: Here there is no direct mention of differential equations, but use of the buzz-phrase *'growing exponentially'* must be taken as indicator that we are talking about the situation

$$f(t)=ce^{kt}$$

where here f(t)f(t) is the number of llamas at time tt and c,kc,k are constants to be determined from the information given in the problem. And the use of language should probably be taken to mean that at time t=0t=0 there are 10001000 llamas, and at time t=4t=4 there are 20002000. Then, either repeating the method above or plugging into the formula derived by the method, we find

c= value of f at t=0 =1000c= value of f at t=0 =1000

$k=\ln f(t_1)-\ln f(t_2)t_1-t_2=\ln 1000-\ln 20000-4$ $k=\ln f(t1)-\ln f(t2)t1-t2=\ln 1000-\ln 20000-4$

$=\ln 10002000-4=\ln_{12}-4=(\ln 2)/4$ $=\ln 10002000-4=\ln 12-4=(\ln 2)/4$

Therefore,

$f(t)=1000e^{\ln 24t}=1000\cdot 2^{t/4}$ $f(t)=1000e\ln 24t=1000\cdot 2t/4$

This is the desired formula for the number of llamas at arbitrary time tt.

Example 2

A colony of bacteria is growing exponentially. At time t=0 it has 10 bacteria in it, and at time t=4 it has 2000. At what time will it have 100,000 bacteria?

Solution: Even though it is not explicitly demanded, we need to find the general formula for the number f(t)f(t) of bacteria at time tt, set this expression equal to 100,000100,000, and solve for tt. Again, we can take a *little* shortcut here since we know that c=f(0) and we are given that f(0)=10 (This is easier than using the bulkier more general formula for finding cc). And use the formula for kk:

$k=\ln f(t_1)-\ln f(t_2)t_1-t_2=\ln 10-\ln 2{,}0000-4=\ln_{102{,}000}-4=\ln 2004$ $k=\ln f(t1)-\ln f(t2)t1-t2=\ln 10-\ln 2{,}0000-4=\ln 102{,}000-4=\ln 2004$

Therefore, we have

$$f(t)=10\cdot e^{\frac{\ln 200}{4}t}=10\cdot 200^{t/4}$$

as the general formula. Now we try to solve

$$100,000=10\cdot e^{\frac{\ln 200}{4}t}$$

for t: divide both sides by the 10 and take logarithms, to get

$$\ln 10,000=\frac{\ln 200}{4}t$$

Thus,

$$t=\frac{4\ln 10,000}{\ln 200}\approx 6.953407835.$$

Exponential Decay Model

Exponential functions can also be used to model populations that shrink (from disease, for example), or chemical compounds that break down over time. We say that such systems exhibit exponential decay, rather than exponential growth. The model is nearly the same, except there is a negative sign in the exponent. Thus, for some positive constant k, we have

$$y=y_0e^{-kt}. \tag{6.8.15}$$

As with exponential growth, there is a differential equation associated with exponential decay. We have

$$y'=-ky_0e^{-kt}=-ky. \tag{6.8.16}$$

Exponential Decay

Systems that exhibit exponential decay behave according to the model

$$y=y_0e^{-kt}, \tag{6.8.17}$$

where y_0 represents the initial state of the system and $k>0$ is a constant, called the decay constant.

Figure 6.8.2 shows a graph of a representative exponential decay function.

Figure 6.8.2 shows a graph of a representative exponential decay function.

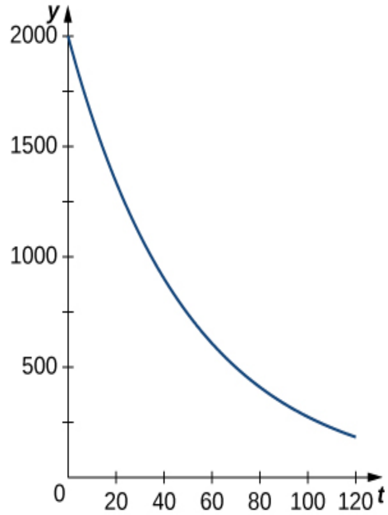


Figure 6.8.2: An example of exponential decay.

Let's look at a physical application of exponential decay. *Newton's law of cooling* says that an object cools at a rate proportional to the difference between the temperature of the object and the temperature of the surroundings. In other words, if T represents the temperature of the object and T_a represents the ambient temperature in a room, then

$$T' = -k(T - T_a). \tag{6.8.18}$$

Note that this is not quite the right model for exponential decay. We want the derivative to be proportional to the function, and this expression has the additional T_a term. Fortunately, we can make a change of variables that resolves this issue. Let $y(t) = T(t) - T_a$. Then $y'(t) = T'(t) - 0 = T'(t)$, and our equation becomes

$$y' = -ky. \tag{6.8.19}$$

From our previous work, we know this relationship between y and its derivative leads to exponential decay. Thus,

$$y = y_0 e^{-kt}, \tag{6.8.20}$$

and we see that

$$T - T_a = (T_0 - T_a)e^{-kt} \tag{6.8.21}$$

$$T = (T_0 - T_a)e^{-kt} + T_a \tag{6.8.22}$$

where T_0 represents the initial temperature. Let's apply this formula in the following example.

Limitations of Law of natural growth or decay

Growth and decay problems are used to determine exponential growth or decay for the general function $f(x) = ka^x$ (for growth, $a > 1$; for decay, $0 < a < 1$). Typical problems involve population, radioactive decay, and Newton's Law of Cooling. They are used to determine the amount of a group after a given starting point. For any population of size *y*, the population changes at a rate proportional to itself, so that the change in population over time is equal to the population size times the rate ($\frac{dy}{dx} = k \times y$, where *k* is a constant). Therefore, all rate problems can be solved using the equation $y(x) = y_0 e^{kx}$ where y_0 is the value of $y(x)$ when $x = 0$ (the original value), *k* is a constant unique to the problem, and *x* is the time interval.

Conclusion: In conclusion, our report has explored the applications of Ordinary Differential Equations (ODEs) in modeling natural phenomena such as growth and decay. Through various examples, such as the exponential growth of a llama population and the expansion of a bacterial colony, we demonstrated how ODEs can predict system dynamics effectively. However, we also acknowledged the models' limitations, emphasizing that real-world conditions could vary and affect outcomes. Despite these constraints, the project underlines the significance of ODEs in understanding and forecasting natural processes, showcasing the intersection of mathematical theory and practical application. This underscores the value of mathematical modeling in solving real-world problems, although it requires careful consideration of each model's assumptions and applicability.

A Field Project Report

on

Applications of ODE-LR Circuits

(MATHEMATICS – FIELD PROJECT)

Submitted by

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March 2023

<u>Certificate</u>

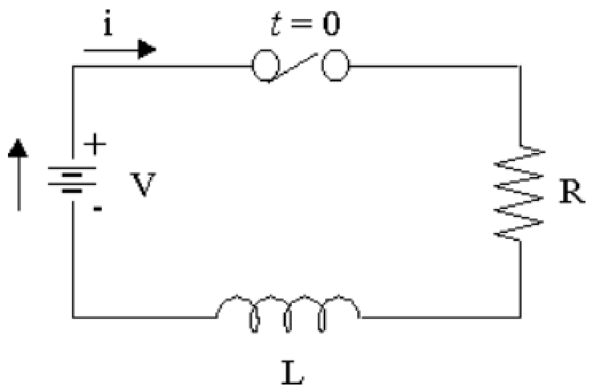
This is to certify the Project report titled **"Applications of ODE - LR Circuits"**submittedby **YERRAKULA LAVANYA (201FM01020), THOTA PURNA RAGAVENDRA BABU (201FM01021), PULIVARTHI SAI GANESH (201FM01022) JANAPANENIKALYANCHAKRAVARTHI(201FM01023), CHUDAMANI KAMJULA (201FM01024)** is carried out as field project work under by supervisor. I approve this field project work for submission towards partial fulfillment of the requirements and course work in prescribed for B. Sc, VFSTR (Deemed to be University)

Project Guide **Head of the Department**

Abstract: Application of Gaussian in circuit theory, using Kirchhoff's 2 nd law. In this paper for a given circuit, forming into matrices form by using Kirchhoff's 2 nd law we solve and find the current values. Less than 3x3 matrices we can use Cramer's rule, but more that 3x3, Cramer's cannot be done, so gauss elimination method is used to find the current values for the given circuits

Introduction: Circuits: An electronic circuit is composed of individual Electronic Components, such as Resistors, Transistors, Capacitors, Inductors and Diodes, connected by conductive wires or traces through which Electric Current can flow. The combination of components and wires allows various simple and complex operations to be performed: signals can be amplified, computations can be performed, and data can be moved from one place to another. Circuits can be constructed of discrete components connected by individual pieces of wire Let R = Resistance of the circuit C = Capacitance in series with R I = Current flowing L = Inductor V R = voltage across R Vc = voltage across

Description: Formation of Ordinary Differential Equations: Series RL Circuit



RL circuit diagram

The RL circuit shown above has a resistor and an inductor connected in series. A constant voltage V is applied when the switch is closed.

RL circuit diagram

The RL circuit shown above has a resistor and an inductor connec
voltage V is applied when the switch is closed.

The (variable) voltage across the **resistor** is given by:

$$V_R = iR$$

The (variable) voltage across the **inductor** is given by:

$$V_L = L\frac{di}{dt}$$

Kirchhoff's voltage law says that the directed sum of the voltages around a circuit must be zero. This results in the following differential equation:

$$Ri + L\frac{di}{dt} = V$$

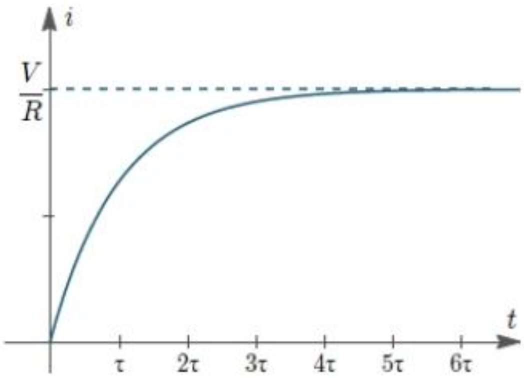
Once the switch is closed, the current in the circuit is not constant. Instead, it will build up from zero to some steady state.

Solving the DE for a Series RL Circuit

The solution of the differential equation $Ri + L\frac{di}{dt} = V$ is:

$$i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$$

Here is the graph of this equation:



Graph of $i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$.

The plot shows the transition period during which the current adjusts from its initial value of zero to the final value $\frac{V}{R}$, which is the **steady state**.

The Time Constant

The **time constant** (TC), known as τ, of the function

$$i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$$

is the time at which $\frac{R}{L}$ is unity (= 1). Thus for the RL transient, the time constant is $\tau = \frac{L}{R}$ seconds.

NOTE: τ is the Greek letter "tau" and is **not** the same as T or the time variable t, even though it looks very similar.

At 1 τ

$1 - e^{-(R/L)t}$

$$= 1 - e^{-1}$$

Solved Examples

Example 1

An RL circuit has an emf of 5 V, a resistance of 50 Ω, an inductance of 1 H, and no initial current.

Find the current in the circuit at any time t. Distinguish between the transient and steady-state current.

Solution:

Method 1 - Solving the DE

The formula is: $Ri + L\frac{di}{dt} = V$

After substituting: $50i + \frac{di}{dt} = 5$

We re-arrange to obtain:

$$\frac{di}{dt} + 50i = 5$$

This is a first order linear differential equation.

We'll need to apply the formula for solving a first-order DE which for these variables will be:

$$ie^{\int P\,dt} = \int \left(Qe^{\int P\,dt}\right) dt$$

We have $P = 50$ and $Q = 5$.

We find the integrating factor:

$\text{I.F.} = e^{\int 50\,dt} = e^{50t}$

So after substituting into the formula, we have:

$$(i)(e^{50t}) = \int (5)e^{50t}\,dt = \frac{5}{50}e^{50t} + K = \frac{1}{10}e^{50t} + K$$

When $t = 0$, $i = 0$, so $K = -\frac{1}{10} = -0.1$.

This gives us: $i = 0.1(1 - e^{-50t})$

The transient current is: $i = 0.1(1 - e^{-50t})$ A.

The steady state current is: $i = 0.1$ A.

Method 2: Using the Formula

NOTE: We can use this formula here only because the voltage is **constant**. This formula wi not work with a variable voltage source.

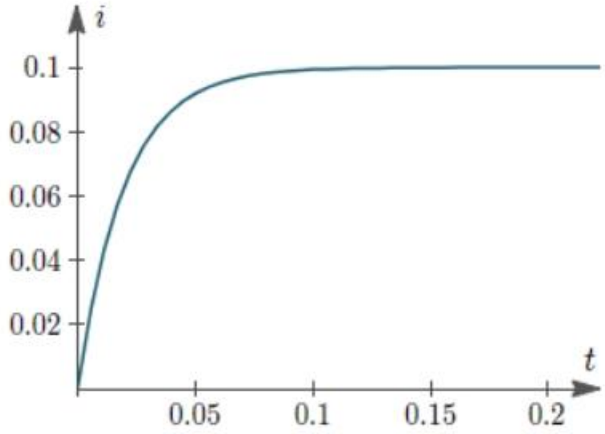
We have the following general formula:

$$i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$$

So in this case:

So in this case:

$$i = \frac{5}{50}\left(1 - e^{-50t}\right) = 0.1\left(1 - e^{-50t}\right)$$



Graph of the current at time t, given by $i = 0.1\left(1 - e^{-50t}\right)$.

In this example, the time constant, TC, is

$$\tau = \frac{L}{R} = \frac{1}{50} = 0.02$$

So we see that the current has reached steady state by $t = 0.02 \times 5 = 0.1$ s.

A series RL circuit with R = 50 Ω and L = 10 H has a constant voltage V = 100 V applied at t = 0 by the closing of a switch.

Find

(a) the equation for i (you may use the formula rather than DE),

(b) the current at t = 0.5 s

(c) the expressions for V_R and V_L

(d) the time at which $V_R = V_L$

Examle:2

A series RL circuit with R = 50 Ω and L = 10 H has a constant voltage V = 100 V applied at t = 0 by the closing of a switch.

Find

(a) the equation for i (you may use the formula rather than DE),

(b) the current at $t = 0.5$ s

(c) the expressions for V_R and V_L

(d) the time at which $V_R = V_L$

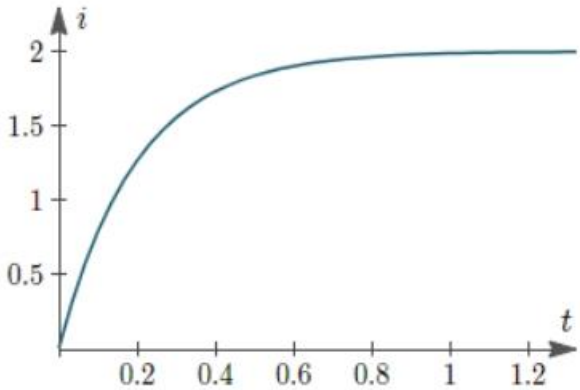
Solution:

(a) We solve it using the formula:

$$i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$$

We have:

$$i = \frac{100}{50}\left(1 - e^{-5t}\right)$$
$$= 2\left(1 - e^{-5t}\right)$$



(b) At $t = 0.5$,

$$i = \left[2\left(1 - e^{-5t}\right)\right]_{t=0.5} = 1.8358$$

(c) V_R and V_L are given by:

$$\begin{aligned} V_R &= iR \\ &= 2\left(1 - e^{-5t}\right) \times 50 \\ &= 100\left(1 - e^{-5t}\right) \\ V_L &= L\frac{di}{dt} \\ &= 10\frac{d}{dt}2\left(1 - e^{-5t}\right) \\ &= 100e^{-5t} \end{aligned}$$

(d) To find the required time, we need to solve when $V_R = V_L$.

$V_R = V_L$ when

$V_R = V_L$ when

$$100\left(1 - e^{-5t}\right) = 100e^{-5t}$$

$$1 - e^{-5t} = e^{-5t}$$

$$2e^{-5t} = 1$$

$$e^{-5t} = 0.5$$

$$-5t = \ln\ 0.5 = -0.69315$$

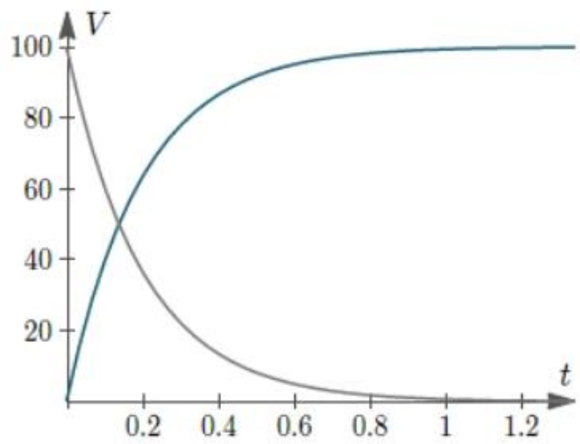
So

$$t = \frac{-0.69315}{-5} = 0.13863s$$

Substituting this value into V_R gives:

$$V_R = V_L = \left[100e^{-5t}\right]_{t=0.13863} = 50.000\ \mathrm{V}$$

The graph of V_R and V_L is as follows:

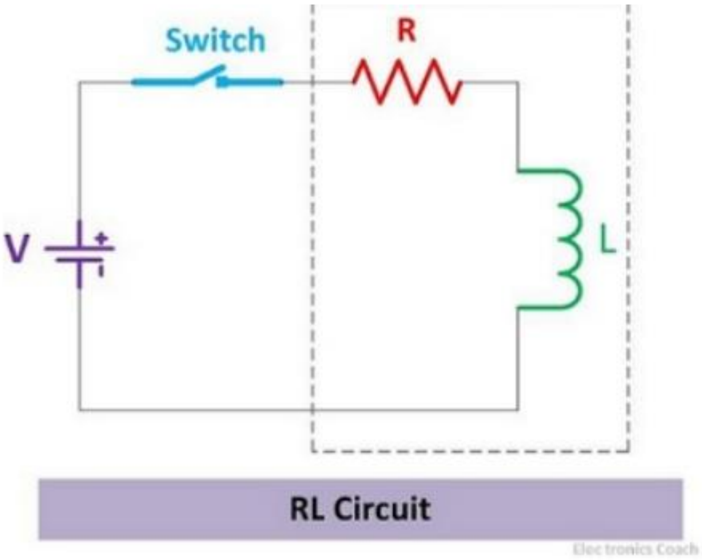


Graph of the voltages $V_R = 100\left(1 - e^{-5t}\right)$ (in green), and $V_L = 100e^{-5t}$ (in gray).

he time constant, TC, for this example is:

$$\tau = \frac{L}{R} = \frac{10}{50} = 0.2$$

Limitations of LR Circuits



1. The RC and RL circuit, both stores energy, but the RC circuit stores energy in the form of an electric field. While RL circuit stores energy in the form of magnetic field.
2. The RC circuits are economical as capacitors are cheap and abundantly available while inductors are costly which makes RL circuit expensive.
3. The inductors possess a wider tolerance ratings in comparison with resistors, thus RL circuit has high tolerance values.
4. The inductor generates the magnetic field which creates noise in the circuit. This leads to poor performance of RL circuit when the noise signal becomes high. The problem of noise can be mitigated by using RC circuit as the capacitor does not generate the magnetic field.

CONCLUSION:

The study of ordinary differential equations in LR circuits reveals their essential role in analyzing the behavior of electrical circuits, particularly in understanding the transient response and time evolution of currents and voltages. By employing ODEs, engineers and physicists can model and predict the behavior of LR circuits accurately, aiding in the design and optimization of electronic systems. This application highlights the significance of mathematical tools in advancing our comprehension and utilization of electrical phenomena.

A Field Project Report

on

Maxima and Minima

(MATHEMATICS – FIELD PROJECT)

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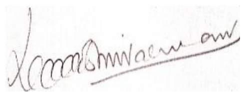


Project Guide **Head of the Department**

Maxima definition:

Let f(x) be a real function defined on an interval I then, f(x) is said to have the maximum value in I, if there exists a point 'a' in I such that $f(x) \leq f(a)$ for all $x \in I$.

The number f(a) is called the maximum value of f(x) in the interval I and point a is called the **point of maxima** of f in interval I. The maxima of a function are defined as the point in the given interval where the function value is maximum. In other words, maxima is the highest point on the curve of a function. There are two types of maxima:

- Local or Relative Maxima
- Absolute or Global Maxima

Minima Definition:

Let f(x) be a real function defined on an interval I then, f(x) is said to have the minimum value in I, if there exists a point 'a' in I such that $f(x) \geq f(a)$ for all $x \in I$.

The number f(a) is called the minimum value of f(x) in interval I and the point a are called the **point of minima** of f in the interval I. The minima of a function is defined as the point in the given interval where the function value is minimum. In other words, minima is the lowest point on the curve of a function.

There are two types of minima:

- Relative or Local Minima
- Absolute or Global Minima

Types of Maxima and Minima

There are two types of maxima and minima. They are listed as follows:

- Relative or Local Maxima and Minima
- Absolute or Global Maxima and Minima

Relative Maxima and Minima:

The relative maxima or relative minima is the maximum and minimum value which is greater than or lesser than its neighbor.

Relative Maxima:

A function f(x) is said to have a relative maximum at x =a if there exists a neighborhood (a – δa, a + δa) of a such that

$f(x) < f(a)$ for all $x \in (a-\delta a, a+\delta a), x \neq a$.

Here, the point a is called the point of relative maxima of a function and f(a) is called as the relative maximum value. The relative maxima is also called as the local maxima of a function.

Relative Minima:

A function f(x) is said to have a relative minimum at x = a if there exists a neighborhood (a-δa, a+δa) of a such that

$f(x) > f(a)$ for all $x \in (a-\delta a, a+\delta a), x \neq a$.

Here, the point a is called the point of minima of a function and f(a) is called as the relative minimum value. The relative minima are also called as the local minima of a function.

Absolute maxima and Absolute minima:

The absolute maxima and the absolute minima are the highest or the lowest value in the entire domain of the function.

Absolute Maxima:

A function f(x) with domain D is said to be absolute maximum at x = a where a ∈ D, if f(x) ≤ f(a) for all x ∈ D. The point a is called the point of absolute maxima of function and f(a) is called as the absolute maximum value. The absolute maxima is also called as the global maxima of a function.

Absolute Minima:

A function f(x) with domain D is said to be absolute maximum at x = a where a ∈ D , if f(x) ≥ f(a) for all x ∈ D. The point a is called the point of absolute maxima of function and f(a) is called as the absolute maximum value. The absolute maxima is also called as the global maxima of a function.

Absolute Maxima	**Relative**

1. What is Maxima and Minima of a Function?

The maximum value of a function at any point is called as maxima of a function and the minimum value of a function at any point is called as minima of a function.

2. What is Point of Inflection?

The stationary point where second order derivative is equal to zero is called as the point of inflection.

3. How to find the Maxima and Minima of a Function?

To find the maxima and minima of a function, we use the first order derivative test and second order derivative test.

Absolute vs Relative Maxima and Minima

The difference between absolute and relative maxima and Minima is tabulated below:

and Minima	Maxima and Minima
It is also called as global maxima or global minima.	It is also called as local maxima or local minima.
It is bounded by domain of the function.	It is not bounded by domain of the function.
It is the highest or lowest point of the function.	It is the higher or lower among both neighbors.
It is the global peak of the curve.	It is the local peak of the curve.

How to Find Maxima and Minima?

We can find maxima and minima of the function by differentiating it. To find the maxima and minima of a function we apply some derivative tests : First Order derivative test and Second Order derivative test. In these tests we differentiate the function a and by checking some conditions we get the maxima and minima point of the function.

Some Derivative Tests:

- First Order Derivative Test
- Second Order Derivative Test.

First Order Derivative Test:

- The first order derivative test as the name suggests it uses first order derivative to find maxima and minima. The first order derivative gives the slope of the function.
- Let f be a continuous function at critical point c on the open interval l such that f'(c) = 0 then, we will check the nature of the curve. Below are some conditions after checking the nature of the curve, and x increases towards c i.e., the critical point.

1.If the sign of f'(x) changes from positive to negative, then f(c) is the maximum value and c is the point of local maxima.

2. If the sign of f'(x) changes from negative to positive, then f(c) is the minimum value and c is the point of local minima.
3. If the sign of f'(x) neither changes from positive to negative nor from negative to positive, then c is called the point of inflection i.e., neither maxima nor minima.

Second Derivative Test

The second order derivative test as the name suggests it uses second order derivative to find maxima and minima.

Condition	**Result**
If f'(c) = 0 and f''(c) < 0	c is the local maxima and f(c) is the maximum value.
If f'(c) = 0 and f''(c) > 0	c is the local minima and f(c) is the minimum value.
If f''(c) = 0	Test fails.

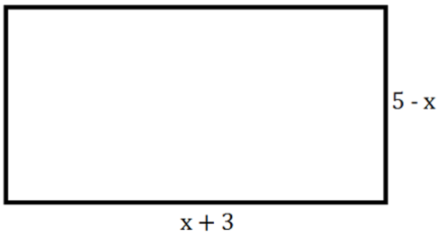
Let f be a function that is two times differentiable at critical point c defined on the open interval. The following are the conditions:

APPLICATIONS:

1. ***Economics:*** In economics, local maxima and minima are used to analyze production functions, cost functions, and utility functions to determine optimal levels of production, pricing, and consumption.
2. ***Engineering:*** In engineering, local maxima and minima are crucial for optimizing designs, such as finding the most efficient shape for a structure or the optimal operating conditions for a system while minimizing costs or maximizing performance.
3. ***Finance:*** In finance, local maxima and minima are used in portfolio optimization, risk management, and option pricing to identify the best investment strategies and to determine when to buy or sell assets.
4. ***Physics:*** In physics, local maxima and minima are applied in various contexts, such as in analyzing potential energy surfaces in chemical reactions, optimizing trajectories in mechanics, and determining stable equilibrium points in systems.
5. ***Machine Learning:*** In machine learning, local maxima and minima play a key role in optimization algorithms used for training models. Gradient descent, for example, is used to find local minima of the loss function to iteratively improve the model's performance.
6. ***Biology:*** In biology, local maxima and minima are used to analyze biological systems, such as enzyme-substrate interactions, population dynamics, and neural network activity, to understand optimal conditions for growth, survival, and function.
7. ***Geography:*** In geography, local maxima and minima are used to analyze terrain features, such as identifying mountain peaks (local maxima) and valleys (local minima), which are essential for navigation, resource management, and environmental studies.
8. ***Computer Vision:*** In computer vision, local maxima and minima are utilized in image processing techniques, such as edge detection and feature extraction, to identify significant points in images for object recognition and analysis.

EXAMPLES:

1. The sides of a rectangular garden are labeled below in feet. Can the area of the garden be made into a maximum area? If so, what is the maximum area?



Solution:

The area of a rectangle is given by the length multiplied by the width. So the area of the garden is given by:

$$A = (x + 3)(5 - x)$$

$$A = -x^2 + 2x + 15$$

We can have a maximum area because *a* is negative. The maximum area is given by:

$$\max = c - \frac{b^2}{4a}$$

$$\max = 15 - \frac{2^2}{4(-1)}$$

$$\max = 15 + 1$$

$$\max = 16 \quad square\ feet$$

2 .Find the local maxima and local minima for the function $y = x^3 - 3x + 2$

Solution:

$$y = x^3 - 3x + 2$$

Find first order derivative.

Differentiating y

$$y' = (d / dx) [x^3 – 3x + 2]$$

$$\Rightarrow y' = (d / dx) x^3 – (d / dx) (3x) + (d / dx) 2$$

$$\Rightarrow y' = 3x^2 – 3 + 0$$

$$\Rightarrow y' = 3x^2 – 3$$

Now equate y' = 0, to find the critical points

$Y' = 0$

$$\Rightarrow 3x^2 – 3 = 0$$

$$\Rightarrow 3x^2 = 3$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = 1 \text{ or } x = -1$$

The critical points are x = 1 and x = -1

Now we will find second derivative to check the critical point is maxima or minima.

$$y'' = (d / dx) [3x^2 – 3]$$

$$\Rightarrow y'' = (d / dx) [3x^2] – (d /dx) [3]$$

$$\Rightarrow y'' = 6x – 0$$

$$\Rightarrow y'' = 6x$$

Now we will put the values of x and find whether y" is greater than 0 or less than 0.

At $x = 1$

$$y'' = 6(1) = 6$$

Since, $y'' > 0$ $x = 1$ *is the minima of* y

At $x = -1$

$$y'' = 6(-1) = -6$$

Since, $y'' < 0$ $x = -1$ *is the maxima of* y

The local maxima and minima of y *are* $x = -1$ *and* $x = 1$ *respectively.*

A Field Project Report

on

Applications of ODE-LRC Circuits

(MATHEMATICS – FIELD PROJECT)

Submitted by

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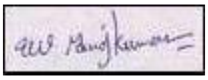
(ACCREDITED BY "NAAC A$^+$" GRADE)

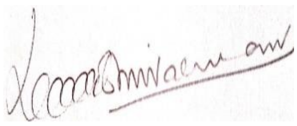
VADLAMUDI, GUNTUR-522213, AP,INDIA

March 2023

Certificate

This is to certify the Project report titled **"Applications of ODE - LRC Circuits"** submitted by **SAMINENI KOUSHIK (201FM01031), THOTA MAHIMANANDA (201FM01032), PUSAPATI SRI SAI NAGA SUPRAJA (201FM01034), AVULA TEJASWINI (201FM01035), SUGGUNA SRINU BABU(201FM01036)** is carried out as field project work under by supervisor. I approve this field project work for submission towards partial fulfillment of the requirements and course work in prescribed for B. Sc, VFSTR (Deemed to be University)





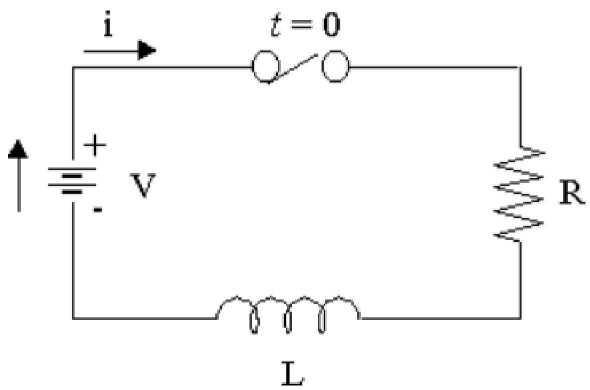
Project Guide

Head of the Department

Abstract: Application of Gaussian in circuit theory, using Kirchhoff's 2 nd law. In this paper for a given circuit, forming into matrices form by using Kirchhoff's 2 nd law we solve and find the current values. Less than 3x3 matrices we can use Cramer's rule, but more that 3x3, Cramer's cannot be done, so gauss elimination method is used to find the current values for the given circuits

Introduction: Circuits: An electronic circuit is composed of individual Electronic Components, such as Resistors, Transistors, Capacitors, Inductors and Diodes, connected by conductive wires or traces through which Electric Current can flow. The combination of components and wires allows various simple and complex operations to be performed: signals can be amplified, computations can be performed, and data can be moved from one place to another. Circuits can be constructed of discrete components connected by individual pieces of wire Let R = Resistance of the circuit C = Capacitance in series with R I = Current flowing L = Inductor V R = voltage across R Vc = voltage across

Description: Formation of Ordinary Differential Equations: Series RL Circuit



RL circuit diagram

The RL circuit shown above has a resistor and an inductor connected in series. A constant voltage *V* is applied when the switch is closed.

RL circuit diagram

The RL circuit shown above has a resistor and an inductor connec
voltage V is applied when the switch is closed.

The (variable) voltage across the **resistor** is given by:

$$V_R = iR$$

The (variable) voltage across the **inductor** is given by:

$$V_L = L\frac{di}{dt}$$

Kirchhoff's voltage law says that the directed sum of the voltages around a circuit must be zero. This results in the following differential equation:

$$Ri + L\frac{di}{dt} = V$$

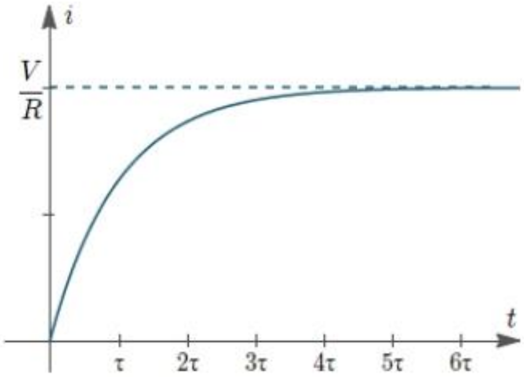
Once the switch is closed, the current in the circuit is not constant. Instead, it will build up from zero to some steady state.

Solving the DE for a Series RL Circuit

The solution of the differential equation $Ri + L\frac{di}{dt} = V$ is:

$$i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$$

Here is the graph of this equation:



Graph of $i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$.

The plot shows the transition period during which the current adjusts from its initial value of zero to the final value $\frac{V}{R}$, which is the **steady state**.

The Time Constant

The **time constant** (TC), known as τ, of the function

$$i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$$

is the time at which $\frac{R}{L}$ is unity (= 1). Thus for the RL transient, the time constant is $\tau = \frac{L}{R}$ seconds.

NOTE: τ is the Greek letter "tau" and is **not** the same as T or the time variable t, even though it looks very similar.

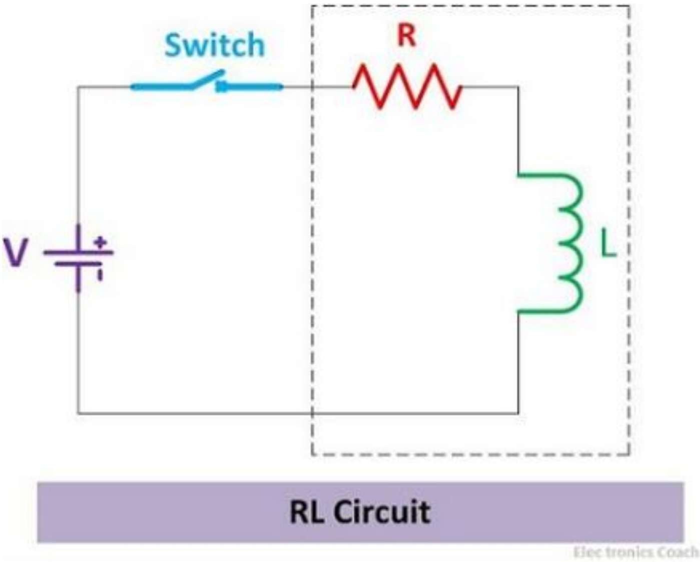
At 1 τ

$1 - e^{-(R/L)t}$

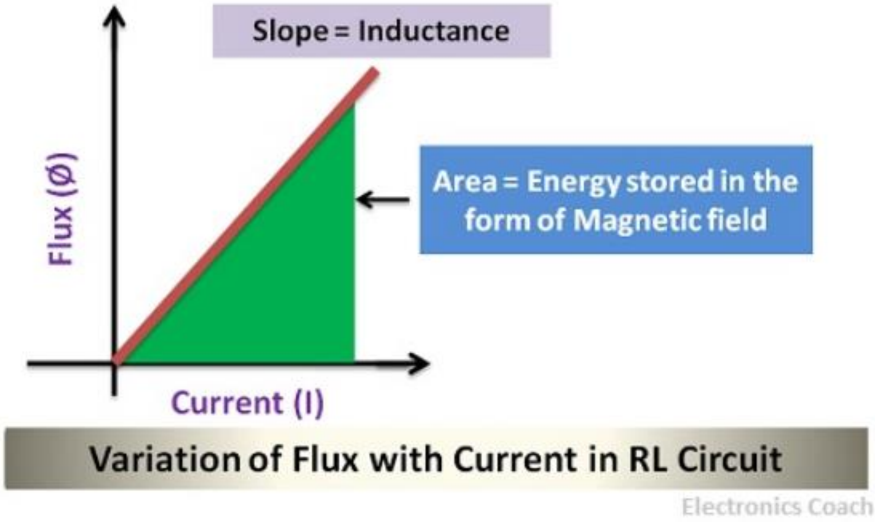
$= 1 - e^{-1}$

RL Circuit (Resistance – Inductance Circuit)

The RL circuit consists of resistance and inductance connected in series with a battery source. The current from the voltage source experiences infinite resistance initially when the switch is closed. As soon as the RL circuit reaches to steady state, the resistance offered by inductor coil begins to decrease and at a point, the value of resistance of RL circuit becomes zero.



The flux linking with the inductor coil creates the magnetic field around it. Moreover, the flux varies with the current flowing through the coil. The variation of flux with the current is shown below with the help of a graph.



The slope of the graph depicts the value of inductance while the area covered by the graph determines the energy stored in the form of magnetic field.

To understand it more clearly, you may refer to the equation shown below. These equations show the relationship between current, inductance and externally supplied voltage with the help of Kirchhoff's law. The energy which is stored in the form of the magnetic field can be expressed in terms of flux and current.

Solved Examples

Example 1

An RL circuit has an emf of 5 V, a resistance of 50 Ω, an inductance of 1 H, and no initial current.

Find the current in the circuit at any time *t*. Distinguish between the transient and steady-state current.

Solution:

Method 1 - Solving the DE

The formula is: $Ri + L\frac{di}{dt} = V$

After substituting: $50i + \frac{di}{dt} = 5$

We re-arrange to obtain:

$$\frac{di}{dt} + 50i = 5$$

This is a first order linear differential equation.

We'll need to apply the formula for solving a first-order DE which for these variables will be:

$$ie^{\int P\,dt} = \int \left(Qe^{\int P\,dt}\right) dt$$

We have $P = 50$ and $Q = 5$.

We find the integrating factor:

$\text{I.F.} = e^{\int 50\,dt} = e^{50t}$

So after substituting into the formula, we have:

$$(i)\left(e^{50t}\right) = \int (5)e^{50t}\,dt = \frac{5}{50}e^{50t} + K = \frac{1}{10}e^{50t} + K$$

When $t = 0$, $i = 0$, so $K = -\frac{1}{10} = -0.1$.

This gives us: $i = 0.1\left(1 - e^{-50t}\right)$

The transient current is: $i = 0.1\left(1 - e^{-50t}\right)\ \text{A}$.

The steady state current is: $i = 0.1\ \text{A}$.

Method 2: Using the Formula

NOTE: We can use this formula here only because the voltage is **constant**. This formula wi not work with a variable voltage source.

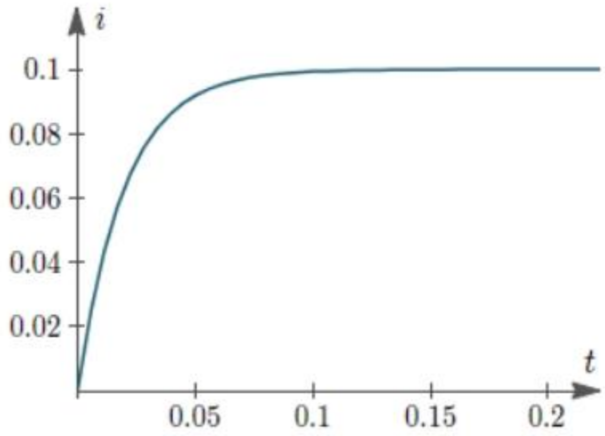
We have the following general formula:

$$i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$$

So in this case:

So in this case:

$$i = \frac{5}{50}\left(1 - e^{-50t}\right) = 0.1\left(1 - e^{-50t}\right)$$



Graph of the current at time t, given by $i = 0.1\left(1 - e^{-50t}\right)$.

In this example, the time constant, TC, is

$$\tau = \frac{L}{R} = \frac{1}{50} = 0.02$$

So we see that the current has reached steady state by $t = 0.02 \times 5 = 0.1$ s.

A series RL circuit with R = 50 Ω and L = 10 H has a constant voltage V = 100 V applied at t = 0 by the closing of a switch.

Find

(a) the equation for i (you may use the formula rather than DE),

(b) the current at t = 0.5 s

(c) the expressions for V_R and V_L

(d) the time at which $V_R = V_L$

Examle:2

A series RL circuit with R = 50 Ω and L = 10 H has a constant voltage V = 100 V applied at t = 0 by the closing of a switch.

Find

(a) the equation for i (you may use the formula rather than DE),

(b) the current at t = 0.5 s

(c) the expressions for V_R and V_L

(d) the time at which $V_R = V_L$

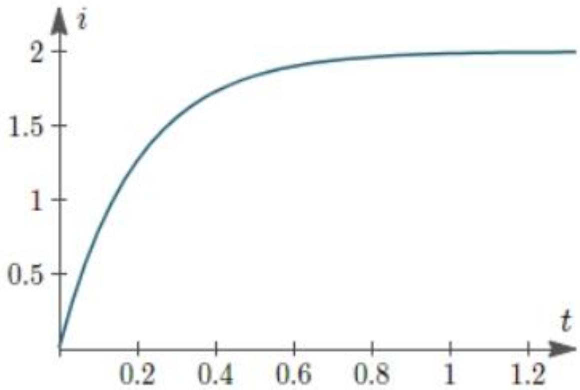
Solution:

(a) We solve it using the formula:

$$i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$$

We have:

$$i = \frac{100}{50}\left(1 - e^{-5t}\right)$$
$$= 2\left(1 - e^{-5t}\right)$$



(b) At $t = 0.5$,

$$i = \left[2\left(1 - e^{-5t}\right)\right]_{t=0.5} = 1.8358$$

(c) V_R and V_L are given by:

$$\begin{aligned} V_R &= iR \\ &= 2\left(1 - e^{-5t}\right) \times 50 \\ &= 100\left(1 - e^{-5t}\right) \\ V_L &= L\frac{di}{dt} \\ &= 10\frac{d}{dt}2\left(1 - e^{-5t}\right) \\ &= 100e^{-5t} \end{aligned}$$

(d) To find the required time, we need to solve when $V_R = V_L$.

$V_R = V_L$ when

$V_R = V_L$ when

$100\left(1 - e^{-5t}\right) = 100e^{-5t}$

$1 - e^{-5t} = e^{-5t}$

$2e^{-5t} = 1$

$e^{-5t} = 0.5$

$-5t = \ln\ 0.5 = -0.69315$

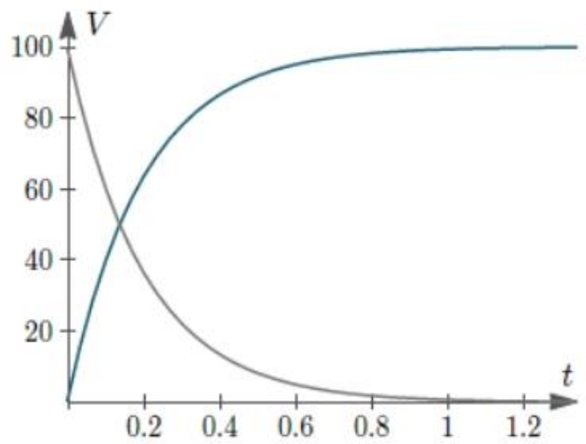
So

$$t = \frac{-0.69315}{-5} = 0.13863s$$

Substituting this value into V_R gives:

$$V_R = V_L = \left[100e^{-5t}\right]_{t=0.13863} = 50.000\ \text{V}$$

The graph of V_R and V_L is as follows:

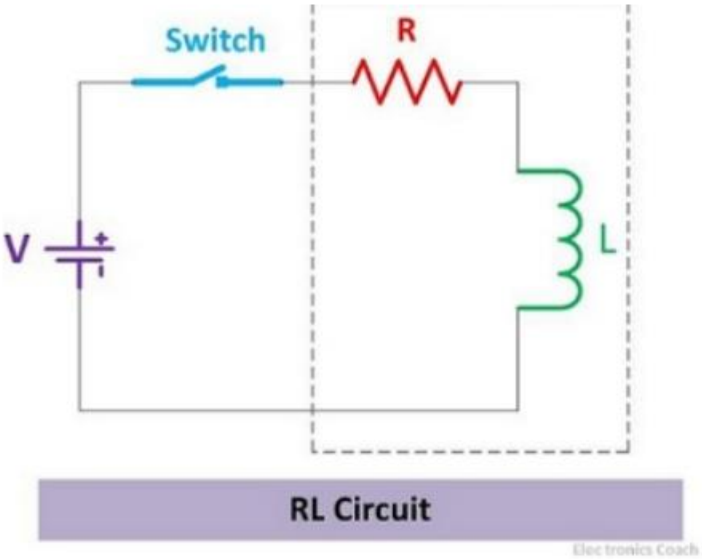


Graph of the voltages $V_R = 100\left(1 - e^{-5t}\right)$ (in green), and $V_L = 100e^{-5t}$ (in gray).

he time constant, TC, for this example is:

$$\tau = \frac{L}{R} = \frac{10}{50} = 0.2$$

Limitations of LR Circuits



1. The RC and RL circuit, both stores energy, but the RC circuit stores energy in the form of an electric field. While RL circuit stores energy in the form of magnetic field.

2. The RC circuits are economical as capacitors are cheap and abundantly available while inductors are costly which makes RL circuit expensive.

3. The inductors possess a wider tolerance ratings in comparison with resistors, thus RL circuit has high tolerance values.

4. The inductor generates the magnetic field which creates noise in the circuit. This leads to poor performance of RL circuit when the noise signal becomes high. The problem of noise can be mitigated by using RC circuit as the capacitor does not generate the magnetic field.

CONCLUSION:

The study of ordinary differential equations in LRC circuits reveals their essential role in analyzing the behavior of electrical circuits, particularly in understanding the transient response and time evolution of currents and voltages. By employing ODEs, engineers and physicists can model and predict the behavior of LR circuits accurately, aiding in the design and optimization of electronic systems. This application highlights the significance of mathematical tools in advancing our comprehension and utilization of electrical phenomena.

A Field Project Report

on

Applications of ODE-Newton's Law of Cooling

(MATHEMATICS – FIELD PROJECT)

Submitted by

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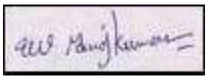
(ACCREDITED BY "NAAC A+" GRADE)

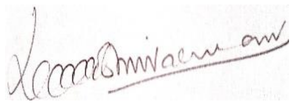
VADLAMUDI, GUNTUR-522213, AP,INDIA

March 2023

Certificate

This is to certify the Project report titled **"Applications of ODE – Newton's Law of Cooling"** submitted by **MAMILLAPALLI DHEERAJ(201FM01037), KAYALA MADHURI(201FM01038), VANUKURI MEGHANA(201FM01039), DRONADULA GOPI KRISHNA (201FM01040), TADANKI VIDYA SAGAR (201FM01041)** is carried out as field project work under by supervisor. I approve this field project work for submission towards partial fulfillment of the requirements and course work in prescribed for B. Sc, VFSTR (Deemed to be University)





Project Guide **Head of the Department**

Abstract: According to Newton's law of cooling, the rate of loss of heat from a body is directly proportional to the difference in the temperature of the body and its surroundings.

INTRODUCTION: Newton's law of cooling describes the rate at which an exposed body changes temperature through radiation which is approximately proportional to the difference between the object's temperature and its surroundings, provided the difference is small.

Newton's law of cooling is given by, $dT/dt = k(T_t – T_s)$

Where,

- T_t = temperature at time t and
- T_s = temperature of the surrounding,
- k = Positive constant that depends on the area and nature of the surface of the body under consideration.

Newton's Law of Cooling Formula

Greater the difference in temperature between the system and surrounding, more rapidly the heat is transferred i.e. more rapidly the body temperature of body changes. Newton's law of cooling formula is expressed by,

$T(t) = T_s + (T_o – T_s)\ e^{-kt}$

Where,

- t = time,
- T(t) = temperature of the given body at time t,
- T_s = surrounding temperature,
- T_o = initial temperature of the body,
- k = constant.

Newton’s Law of Cooling Derivation

For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

$dQ/dt \propto (q - q_s)]$, where q and q_s are temperature corresponding to object and surroundings.

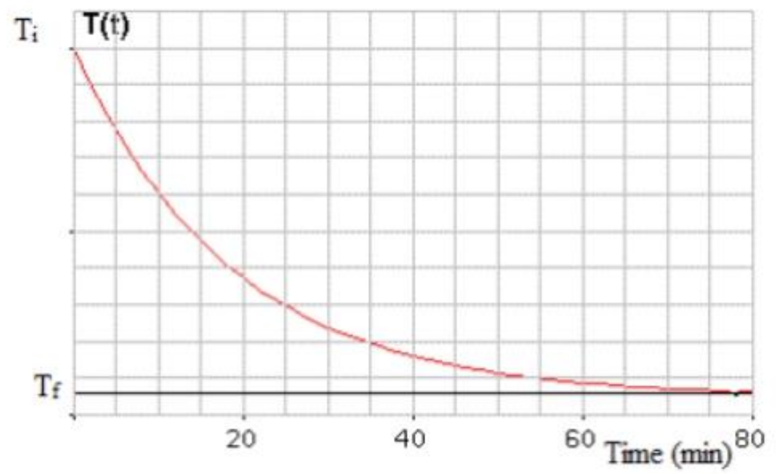
From above expression , $dQ/dt = -k[q - q_s)]$ . . . . (1)

This expression represents Newton’s law of cooling. It can be derived directly from stefan’s law, which gives,

$k = [4e\sigma\times\theta^3_o/mc]\ A$ . . . . . (2)

Now, $d\theta/dt = -k[\theta - \theta_o]$

$$\Rightarrow \int_{\theta_1}^{\theta_2} \frac{d\theta}{(\theta-\theta_o)} = \int_0^1 -kdt$$



Newton’s Law of Cooling – Temperature vs Time

where,

q_i = initial temperature of object,

q_f = final temperature of object.

$\ln (q_f - q_0)/(q_i - q_0) = kt$

$(q_f - q_0) = (q_i - q_0)\ e^{-kt}$

$q_f = q_0 + (q_i - q_0)\ e^{-kt}$ . . . . (3).

Methods to Apply Newton’s Law of Cooling

Sometime when we need only approximate values from Newton’s law, we can assume a constant rate of cooling, which is equal to the rate of cooling corresponding to the average temperature of the body during the interval.

i.e. dθ/dt = k(<q> – q_0) . . . . (4)

If q_i and q_f be the initial and final temperature of the body then,

<q> = (q_i + q_f)/2 . . . . . (5)

Remember equation (5) is only an approximation and equation (1) must be used for exact values.

Solved Examples

Example 1: A body at temperature 40ºC is kept in a surrounding of constant temperature 20ºC. It is observed that its temperature falls to 35ºC in 10 minutes. Find how much more time will it take for the body to attain a temperature of 30ºC.

Solution:

From Newtons law of cooling, $q_f = q_i e^{-kt}$

Now, for the interval in which temperature falls from 40 to 35ºC.

$(35 – 20) = (40 – 20) e^{-k.10}$

$e^{-10k} = 3/4$

k = [ln 4/3]/10 . . . . (a)

Now, for the next interval;

$(30 – 20) = (35 – 20)e^{-kt}$

$e^{-kt} = 2/3$

kt = ln 3/2 . . . . (b)

From equation (a) and (b);

t = 10 × [ln(3/2)/ln(4/3)]= 14.096 min.

Aliter : (by approximate method)

For the interval in which temperature falls from 40 to 35ºC

<q> = (40 + 35)/2 = 37.5ºC

From equation (4);

dθ/dt = k(<q> – q_0)

(35 – 40)/10 = k(37.5 – 20)

k = 1/32 min^{-1}

Now, for the interval in which temperature falls from 35ºC to 30ºC

<q> = (35 + 30)/2 = 32.5ºC

From equation (4);

(30 – 35)/t = (32.5 – 20)

Therefore, the required time t = 5/12.5 × 35 = 14 min.

Example 2: The oil is heated to 70ºC. It cools to 50ºC after 6 minutes. Calculate the time taken by the oil to cool from 50ºC to 40ºC given the surrounding temperature T_s = 25ºC.

Solution:

Given:

Temperature of oil after 10 min = 50ºC,

- T_s = 25ºC,

- $T_o = 70^oC$,
- $t = 6$ min

On substituting the given data in Newton's law of cooling formula, we get;

$T(t) = T_s + (T_s – T_o)\ e^{-kt}$

$[T(t) – T_s]/[T_o – T_s] = e^{-kt}$

$-kt \ln = [\ln T(t) – T_s]/T_o – T_s$

$-kt = [\ln 50 – 25]/70 – 25 = \ln 0.555$

$k = – (-0.555/6) = 0.092$

If $T(t) = 45^oC$ (average temperature as the temperature decreases from 50^oC to 40^oC)

Time taken is $-kt \ln e = [\ln T(t) – T_s]/[T_o – T_s]$

$-(0.092)\ t = \ln 45 – 25/[70 – 25]$

$-0.092\ t = -0.597$

$t = -0.597/-0.092 = 6.489$ min.

Example 3: Water is heated to 80^oC for 10 min. How much would be the temperature if k = 0.056 per min and the surrounding temperature is 25^oC?

Solution:

Given:

- $T_s = 25^oC$,
- $T_o = 80^oC$,
- $t = 10$ min,
- $k = 0.056$

Now, substituting the above data in Newton's law of cooling formula,

$T(t) = T_s + (T_o – T_s) \times e^{-kt}$

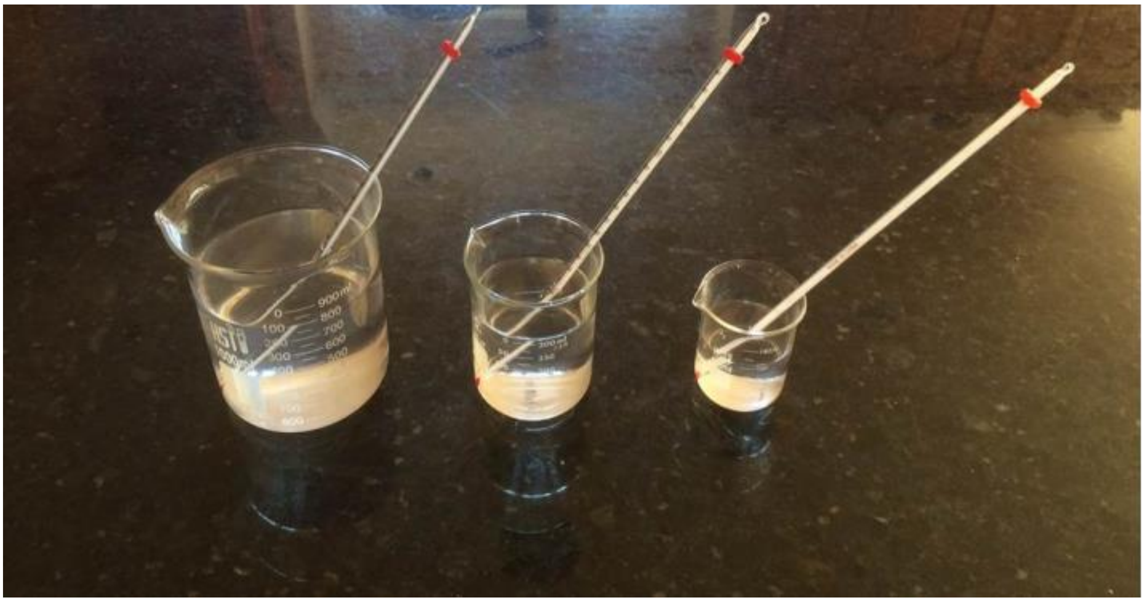
$= 25 + (80 – 25) \times e^{-0.56} = 25 + [55 \times 0.57] = 45.6\ ^oC$

Temperature cools down from 80^oC to 45.6^oC after 10 min.

Experimental Investigation

For this exploration, Newton's Law of Cooling was tested experimentally by measuring the temperature in three beakers of water as they cooled from boiling. The purpose of this investigation was twofold. First I wanted to determine how well Newton's law of cooling fit real data. Second, I wanted to investigate the effect of changing the volume of water being cooled.

Three beakers of water were used for this experiment. The first held 100 ml of water, the second 300 ml, and the third 800 ml. All three beakers originally held water at 100°C. Each beaker had its own thermometer and the thermometers were kept in the beakers between measurements so there would be no temperature lag. The temperature of the water in each beaker was measured every minute, always in the same order. The ambient temperature for this investigation was 23°C. The experimental setup is shown below.



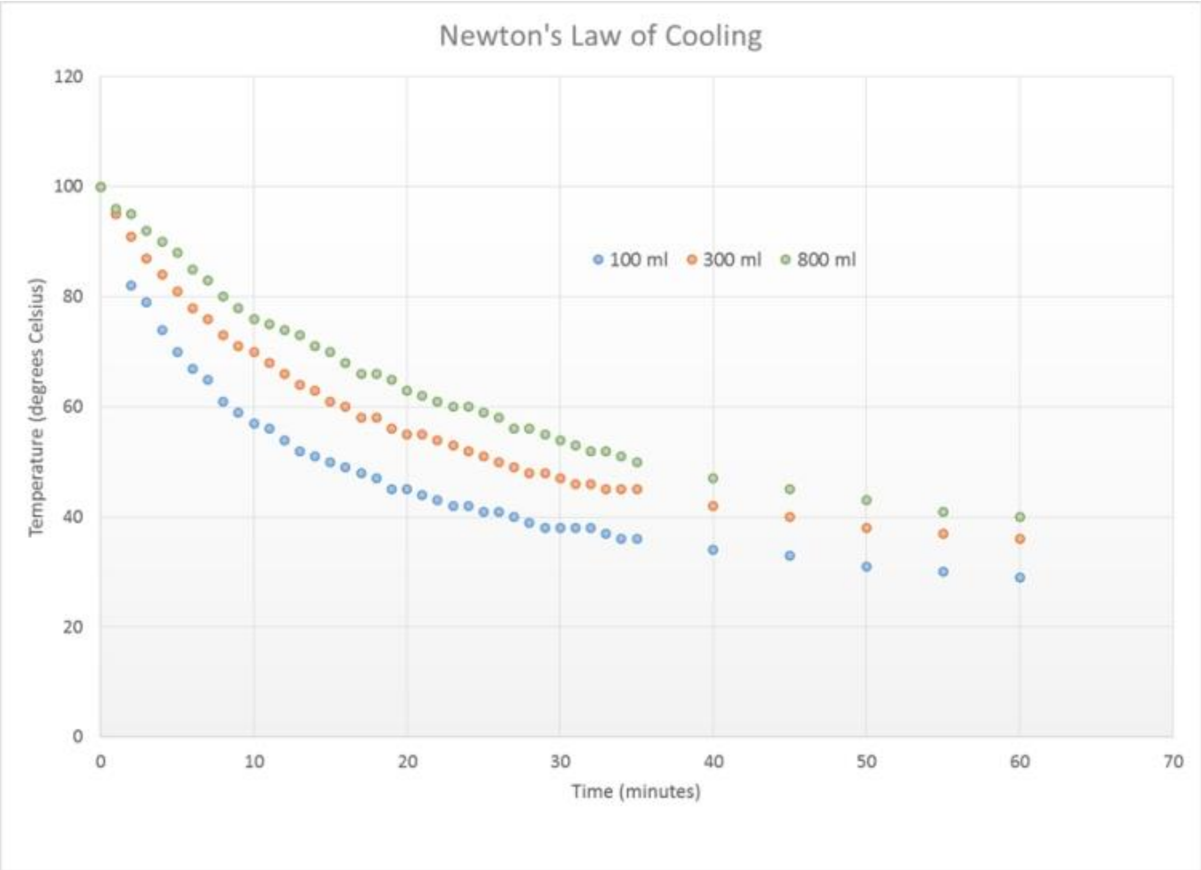
Conclusion: The temperature was measured every minute for 35 minutes and then every 5 minutes for the remainder of one hour. The following data was obtained.

Time	100 ml	300 ml	800 ml

(min)	Temperature °C	Temperature °C	Temperature °C
0	100	100	100
1	95	95	96
2	82	91	95
3	79	87	92
4	74	84	90
5	70	81	88
6	67	78	85
7	65	76	83
8	61	73	80
9	59	71	78
10	57	70	76
11	56	68	75
12	54	66	74
13	52	64	73
14	51	63	71
15	50	61	70
16	49	60	68
17	48	58	66
18	47	58	66
19	45	56	65
20	45	55	63
21	44	55	62
22	43	54	61
23	42	53	60
24	42	52	60
25	41	51	59
26	41	50	58
27	40	49	56
28	39	48	56

29	38	48	55
30	38	47	54
31	38	46	53
32	38	46	52
33	37	45	52
34	36	45	51
35	36	45	50
40	34	42	47
45	33	40	45
50	31	38	43
55	30	37	41
60	29	36	40

From this data, it can be observed that the water in the smaller beakers cooled more quickly than the water in the larger beakers. Below is a graph of the data.



In all of these cases, the experimental temperature fell more quickly at the beginning of the experiment than that predicted by the theoretical model and more slowly than predicted toward the end. The larger water sample followed the Newton's Law of Cooling model more closely than the smaller samples did. There are several explanations for this from a thermodynamics standpoint. Newton's Law of Cooling accounts primarily for conductive heat exchange and assumes that the only heat lost by the system to the surroundings is that due to the temperature difference. At temperatures near boiling, the rate of evaporation is high. The heat lost through the phase change is greater than the heat lost through convective heat exchange with the environment. Additionally, since the beakers were placed on a granite countertop, the heat lost through conduction with the countertop at the beginning of the experiment is significant and is higher than later on when the countertop has warmed up. If the countertop is now warmer than the surrounding air, the temperature gradient is not what it was assumed to be from the initial temperature measurement. Despite these complications, we conclude that Newton's Law of Cooling provides a reasonable approximation of the change in temperature for an object cooling in a constant ambient temperature.

Limitations of Newtons Law of Cooling

- The difference in temperature between the body and surroundings must be small,
- The loss of heat from the body should be by radiation only,
- The major limitation of Newton's law of cooling is that the temperature of surroundings must remain constant during the cooling of the body.

Conclusion:

ODE problems based on Newton's Law of Cooling have widespread applications in diverse fields, ranging from engineering and medicine to environmental science and forensics. The modeling and analysis of cooling rates are fundamental in optimizing processes, ensuring safety, and advancing our understanding of dynamic systems influenced by temperature changes.